EE 2351 – Circuits II
Test # 2

KEY 2

Jul. 24, 2017

Instructions:

a. **Please keep your cell phone stored in your bag or pocket. No cellphone access during the exam. If you are found using your cellphone, you will be asked to leave the room and will receive a grade of 0 in the test.**

b. **You cannot talk to your classmates during the exam. If you talk to your classmates during the exam, you will be asked to leave the room and will receive a grade of 0 in the test.**

c. **This is a closed book, closed notes, no computer exam. All the formulas are provided for the test in the last page of this exam. DO NOT TEAR ANY PAGES.**

d. **Put the proper units and prefixes with your answers and use the appropriate sign conventions.**

e. **Show all work, including intermediate steps. Failure to do so will be penalized. Explicitly state in your answer when the calculator is used to get the roots of a polynomial or to solve a system of equations.**

f. **Write clearly the answer(s) to each question and highlight them or box them. Do all your work on the pages provided. No scrap paper is permitted. You may also use the back of the paper if you run out of space.**

g. **No bathroom breaks during the exam.**

By signing this exam, you agree that the work presented here represents only your effort.

Name: KEY **Signature:** KEY

UTEP ID: KEY

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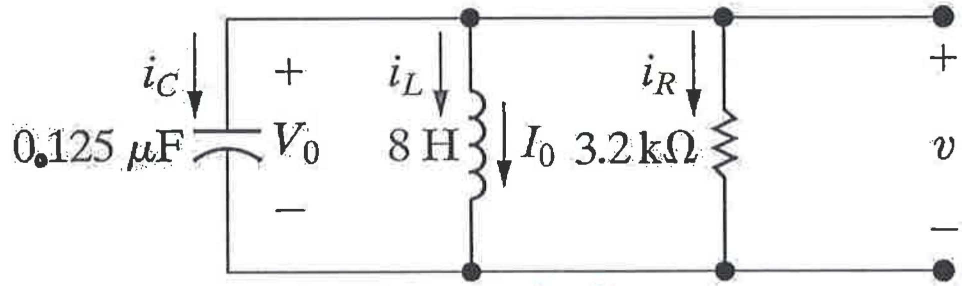
1.– Natural/Step Response of Parallel RLC Circuits (30 points)

In the circuit shown below, $V_0 = 0$, and $I_0 = -12.25$ mA.

Find $v(t)$ when $t \geq 0$.



Natural Response of Parallel RLC

$\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(3200)(0.125\times10^{-6})} = \boxed{1250 = \alpha}$

$\alpha^2 > \omega_0^2$ $\boxed{\text{Overdamped}}$

$\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{8\,(0.125\times10^{-6})}} = \boxed{1000 = \omega_0}$

$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$, $x(0) = A_1 + A_2$

$\dfrac{dx(0)}{dt} = A_1 s_1 + A_2 s_2$

$s_1 = -1250 + \sqrt{(1250)^2 - (1000)^2}$

$\boxed{s_1 = -500}$

$s_2 = -1250 - 750$

$\boxed{s_2 = -2000}$

$V_0 = v(0^+) = \boxed{0 = A_1 + A_2}$

$\dfrac{dv(0^+)}{dt} = \dfrac{i_C(0^+)}{C} = \dfrac{-(-12.25\times10^{-3})}{0.125\times10^{-6}} = 98{,}000\,\dfrac{V}{s}$

$\boxed{98{,}000 = -500A_1 - 2000A_2}$

solve for A_1

$-A_1 = +A_2 \rightarrow 98{,}000 = -500A_1 - 2000(-A_1)$

$98{,}000 = 1500A_1 \rightarrow \boxed{A_1 = 65.33}$

$\boxed{A_2 = -A_1 = -65.33}$

So $\boxed{v(t) = 65.33\left(e^{-500t} - e^{-2000t}\right)\text{ V},\ t \geq 0}$ ← 2

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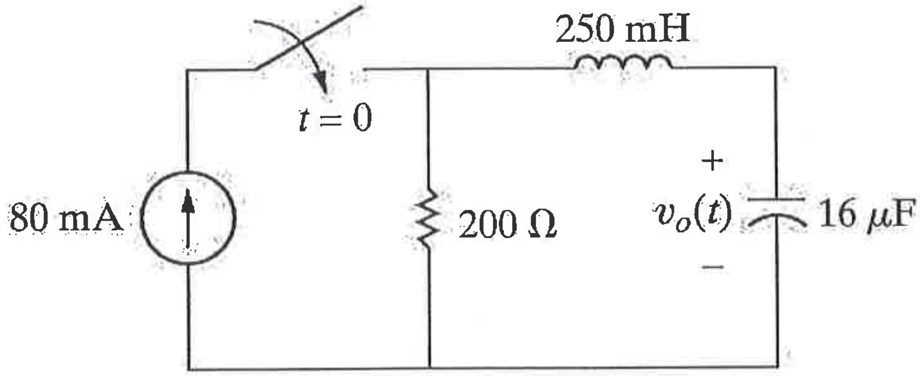
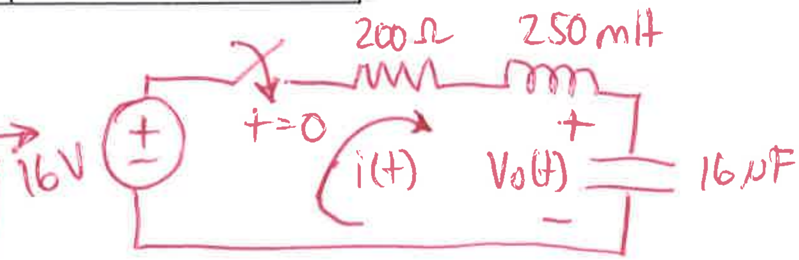
2. – Natural/Step Response of Series RLC Circuits (30 points)

The initial energy stored in the circuit shown below is zero. Find $v_0(t)$ when $t \geq 0$.



Step Response of Series RLC:

Do source transformation →



$\boxed{\alpha = \frac{R}{2L}} = \frac{200}{2(0.25)} = \boxed{400 = \alpha}$

not provided on test
not penalized

$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} = 500 \frac{rad}{s}}$

$\alpha^2 < \omega_0^2$ underdamped

$\omega_d = \sqrt{500^2 - 400^2} = \boxed{300 \frac{rad}{s} = \omega_d}$

$V_o(t) = V_F + B_1 e^{-s_1 t}\cos(\omega_d t) + B_2 e^{-s_2 t}\sin(\omega_d t)$

Find V_F → as $t \to \infty$, $V_o(\infty) = \boxed{16\text{ V (open ckt)}}$

$V_o(0^+) = B_1 + V_F$ → since no energy is stored, $V_o(0^+) = 0$
also capacitor prevents instantaneous change in V

$\boxed{B_1 = 0 - 16}$

$\boxed{B_1 = -16}$

$\frac{dV_o(0^+)}{dt} = -\alpha B_1 + \omega_d B_2$

again, capacitor is discharged and prevents sudden voltage changes, so $\frac{dV(0^+)}{dt} = 0$

$0 = -\alpha B_1 + \omega_d B_2$

$B_2 = \frac{\alpha B_1}{\omega_d} = \frac{400(-16)}{300} = \boxed{-21.33 = B_2}$

So $\boxed{V_o(t) = 16 - 16e^{-400t}\cos(300t) - 21.33e^{-400t}\sin(300t)}$

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3. – Inverse Laplace Transform (20 points)

Find the next parts for the following function:

$$F(s) = \frac{20s^2 + 141s + 315}{s(s^2 + 10s + 21)}$$

a) How many roots does this function have? (5 points)
b) Find the K coefficient for each root using Partial Fraction Expansion. (10 points)
c) Find the Inverse Laplace Function $f(t)$. (5 points)

a) $\boxed{3 \text{ roots}}$

b) Factor $s^2+10s+21 \longrightarrow (s+3)(s+7)$

$$F(s) = \frac{20s^2+141s+315}{s(s+3)(s+7)} = \frac{K_1}{s} + \frac{K_2}{s+3} + \frac{K_3}{s+7}$$

$$K_1 = \left.\frac{20s^2+141s+315}{(s+3)(s+7)}\right|_{s=0} = \frac{315}{21} = \boxed{15 = K_1}$$

$$K_2 = \left.\frac{20s^2+141s+315}{(s+7)(s)}\right|_{s=-3} = \frac{180-423+315}{-12} = \boxed{-6 = K_2}$$

$$K_3 = \left.\frac{20s^2+141s+315}{(s+3)(s)}\right|_{s=-7} = \frac{980-987+315}{28} = \boxed{11 = K_3}$$

c) By using the tables,

$$\boxed{f(t) = \mathcal{L}^{-1}\{F(s)\} = (15 - 6e^{-3t} + 11e^{-7t})u(t)}$$

KEY

4. – Inverse Laplace Transform (20 points)

Find $f(t)$ for the following function:

$$F(s) = \frac{-s^2 + 52s + 445}{s(s^2 + 10s + 89)}$$

a) How many roots does this function have? (5 points)
b) Find the K coefficient for each root using Partial Fraction Expansion. (10 points)
c) Find the Inverse Laplace Function $f(t)$. (5 points)

a) $\boxed{3 \text{ roots}}$

b) Factor $s^2+10s+89 \rightarrow (s+5-j8)(s+5+j8)$

$$F(s) = \frac{-s^2+52s+445}{s(s+5-j8)(s+5+j8)} = \frac{K_1}{s} + \frac{K_2}{s+5-j8} + \frac{K_3}{s+5+j8}$$

$$K_1 = \left.\frac{-s^2+52s+445}{(s+5-j8)(s+5+j8)}\right|_{s=0} = \frac{445}{89} = \boxed{5 = K_1}$$

$$K_2 = \left.\frac{-s^2+52s+445}{s(s+5+j8)}\right|_{s=-5+j8} = \frac{224+j496}{-188-j80} = \boxed{-3-j2 = K_2} \quad \boxed{K_2 = 3.6\angle -146.31^\circ}$$

$$\boxed{K_3 = K_2^* = 3.6\angle +146.31^\circ = -3+j2}$$

c) By using the table: $2|K|e^{-\alpha t}\cos(\beta t + \theta^\circ)u(t)$

$\boxed{2|K| = 7.2, \quad \alpha = +5, \quad \beta = 8}$

$$\boxed{f(t) = \mathcal{L}^{-1}\{F(s)\} = [5 + 7.2e^{-5t}\cos(8t - 146.31^\circ)]u(t)}$$

TABLE 8.1 Natural Response Parameters of the Parallel *RLC* Circuit

Parameter	Terminology	Value In Natural Response
s_1, s_2	Characteristic roots	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
α	Neper frequency	$\alpha = \frac{1}{2RC}$
ω_0	Resonant radian frequency	$\omega_0 = \frac{1}{\sqrt{LC}}$

TABLE 8.2 The Response of a Second-Order Circuit is Overdamped, Underdamped, or Critically Damped

The Circuit is	When	Qualitative Nature of the Response
Overdamped	$\alpha^2 > \omega_0^2$	The voltage or current approaches its final value without oscillation
Underdamped	$\alpha^2 < \omega_0^2$	The voltage or current oscillates about its final value
Critically damped	$\alpha^2 = \omega_0^2$	The voltage or current is on the verge of oscillating about its final value

TABLE 8.3 In Determining the Natural Response of a Second-Order Circuit, We First Determine Whether it is Over-, Under-, or Critically Damped, and Then We Solve the Appropriate Equations

Damping	Natural Response Equations	Coefficient Equations
Overdamped	$x(t) = A_1e^{s_1t} + A_2e^{s_2t}$	$x(0) = A_1 + A_2$; $dx/dt(0) = A_1s_1 + A_2s_2$
Underdamped	$x(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t)e^{-\alpha t}$	$x(0) = B_1$; $dx/dt(0) = -\alpha B_1 + \omega_d B_2$, where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Critically damped	$x(t) = (D_1t + D_2)e^{-\alpha t}$	$x(0) = D_2$, $dx/dt(0) = D_1 - \alpha D_2$

TABLE 8.4 In Determining the Step Response of a Second-Order Circuit, We Apply the Appropriate Equations Depending on the Damping

Damping	Step Response Equations[a]	Coefficient Equations
Overdamped	$x(t) = X_f + A_1'e^{s_1t} + A_2'e^{s_2t}$	$x(0) = X_f + A_1' + A_2'$; $dx/dt(0) = A_1's_1 + A_2's_2$
Underdamped	$x(t) = X_f + (B_1' \cos \omega_d t + B_2' \sin \omega_d t)e^{-\alpha t}$	$x(0) = X_f + B_1'$; $dx/dt(0) = -\alpha B_1' + \omega_d B_2'$
Critically damped	$x(t) = X_f + D_1' te^{-\alpha t} + D_2' e^{-\alpha t}$	$x(0) = X_f + D_2'$; $dx/dt(0) = D_1' - \alpha D_2'$

[a] where X_f is the final value of $x(t)$.

TABLE 12.1 An Abbreviated List of Laplace Transform Pairs

Type	$f(t)\ (t > 0-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

TABLE 12.2 An Abbreviated List of Operational Transforms

Operation	$f(t)$	$F(s)$
Multiplication by a constant	$Kf(t)$	$KF(s)$
Addition/subtraction	$f_1(t) + f_2(t) - f_3(t) + \cdots$	$F_1(s) + F_2(s) - F_3(s) + \cdots$
First derivative (time)	$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
Second derivative (time)	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$
nth derivative (time)	$\frac{d^nf(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{df^2(0^-)}{dt^2} - \cdots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$
Time integral	$\int_0^t f(x)\,dx$	$\frac{F(s)}{s}$
Translation in time	$f(t-a)u(t-a),\ a > 0$	$e^{-as}F(s)$
Translation in frequency	$e^{-at}f(t)$	$F(s+a)$
Scale changing	$f(at),\ a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
First derivative (s)	$tf(t)$	$-\frac{dF(s)}{ds}$
nth derivative (s)	$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
s integral	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$

TABLE 12.3 Four Useful Transform Pairs

Pair Number	Nature of Roots	$F(s)$	$f(t)$
1	Distinct real	$\frac{K}{s + a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{K}{(s + a)^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{K}{s + \alpha - j\beta} + \frac{K^*}{s + \alpha + j\beta}$	$2\|K\|e^{-\alpha t}\cos(\beta t + \theta)u(t)$
4	Repeated complex	$\frac{K}{(s + \alpha - j\beta)^2} + \frac{K^*}{(s + \alpha + j\beta)^2}$	$2t\|K\|e^{-\alpha t}\cos(\beta t + \theta)u(t)$

Note: In pairs 1 and 2, K is a real quantity, whereas in pairs 3 and 4, K is the complex quantity $|K| \angle\theta$.