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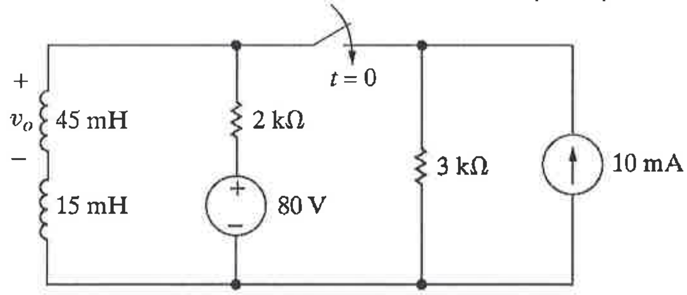
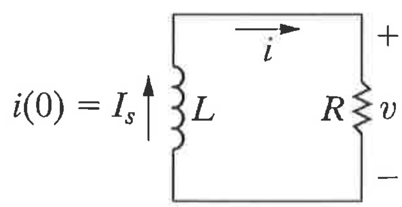
Short Answer Questions (20 pts)

1. Circuit Order: Is this circuit 1[st] or 2[nd] order? Yes or no and explain your answer.



1st order - 2 inductors in series can be replaced by $L_{eq} = L_1 + L_2 = 60\, mH$

2. Time Constants in First-Order Circuits: For the circuit shown below, we showed in class that $i(t) = I_s e^{-t/\tau}$, for t≥0. Approximately, how many time constants, τ, does is take for the inductor current to be at 2% of the initial current value? Approximately, how many time constants does it take for the resistor to dissipate 98% percent of the initial energy stored in the inductor?



$i(4\tau) \approx 0.02\, I_s \Rightarrow$ 4 time constants for the inductor current

$w_R(t) = \frac{1}{2} L I_s^2 (1 - e^{-2t/\tau})$

$w_R(2\tau) \approx \frac{1}{2} L I_s^2 (1 - e^{-4}) \approx 0.98 \frac{1}{2} L I_s \Rightarrow$ 2 time constants for the energy

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3. RLC Damping: In a series RLC circuit, for fixed values of L and C, how does changing the value of R changes the damping of the circuit?

$$\alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$\alpha > \omega_0$ - overdamped

$\alpha < \omega_0$ - underdamped

$\alpha = \omega_0$ - critically damped

$R = R_{crit} = 2\sqrt{\frac{L}{C}}$ critical damping

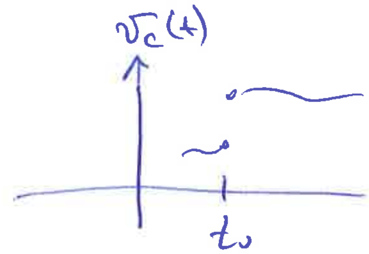
$$\frac{R_{crit}}{2L} = \frac{1}{\sqrt{LC}} \Rightarrow R_{critical} = 2\sqrt{\frac{L}{C}}$$

$\Rightarrow R > 2\sqrt{\frac{L}{C}}$ - overdamped

$R < 2\sqrt{\frac{L}{C}}$ underdamped

4. Continuity of the Capacitor Voltage: Give a short explanation of why the voltage across a capacitor cannot change instantaneously or equivalently it is continuous.

$$v_c(t_0^-) = v(t_0^+)$$



If $v_c(t_0^-) \neq v(t_0^+)$ then

that will imply that $\frac{dv_c}{dt}(t_0) = \infty$

Note that

$$w_c = \frac{1}{2} C v_c^2 \Rightarrow p_c = C v_c \frac{dv_c}{dt}$$

If $\frac{dv_c}{dt} = \infty \rightarrow p_c = \infty \Rightarrow$ infinite power is not physically possible.

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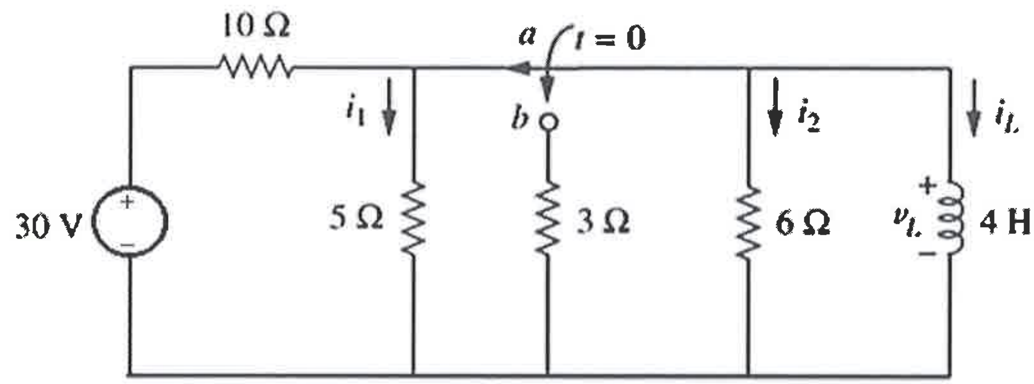
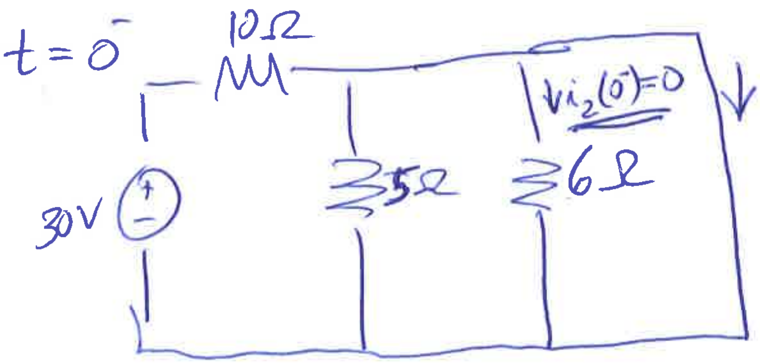
5. For the circuit shown below, the switch is in position **a** for a long time before changing to position **b** at t=0. **(40 points)**



   a. Find $i_2(0^-)$ and $i_2(0^+)$.
   b. Is $i_2(0^-) = i_2(0^+)$? Answer yes or no and explain your answer.
   c. Determine the time constant of the circuit for t>0.
   d. Write the expression for $i_L(t)$ for t≥0.
   e. Find the energy stored in the 4H inductor at t=0?
   f. What percentage of the initial energy stored in the inductor is dissipated in the 6 Ω resistor?

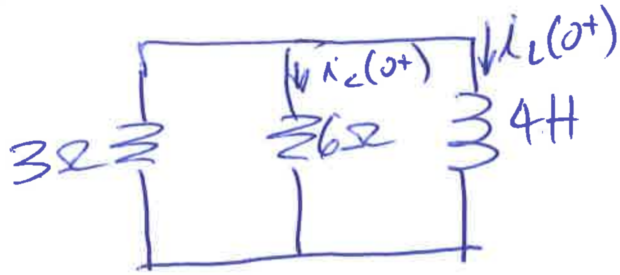
(a) $t=0^-$



$i_L(0^-) = \frac{30}{10} = 3A$

$i_2(0^-) = 0$

$t=0^+$



$i_L(0^+) = i_L(0^-) = 3A$

$i_2(0^+) = \frac{-3}{9}(3) = -1A$

(b) $\Rightarrow i_2(0^-) \neq i_2(0^+) \rightarrow$ current through a resistor is not continuous.

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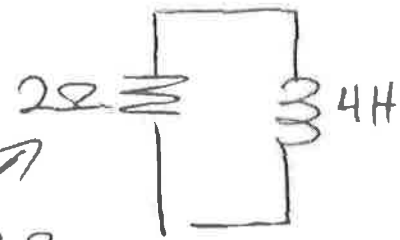
(c)



$\frac{(3)(6)}{9} = 2\Omega$

$\tau = \frac{L}{R} = \frac{4}{2} = 2 \text{ sec}$

(d) $i_L(t) = I_0 e^{-t/\tau}, \; t \geq 0$

$i_L(t) = 3 e^{-t/2}, \; t \geq 0$

(e) $w_L(0) = \frac{1}{2} L i_L^2(0) = \frac{1}{2} 4 (3)^2 = 18 \text{ J}$

(f) $i_2(t) = -\frac{3}{9} i_L(t) = -\frac{1}{3}\left(3e^{-t/2}\right) = -e^{-t/2}, \; t \geq 0$

$P_{6\Omega}(t) = i_2^2 R = 6 e^{-t} \quad t \geq 0$

$W_{6\Omega} = \int_0^{\infty} 6 e^{-t} dt = -6 e^{-t} \Big|_0^{\infty} = 6 \text{ J}$

$\% = \frac{W_{6\Omega}}{w_L(0)} = \frac{6}{18} = \frac{1}{3} \Rightarrow 33\%$ of the total energy is dissipated in the 6Ω resistor.

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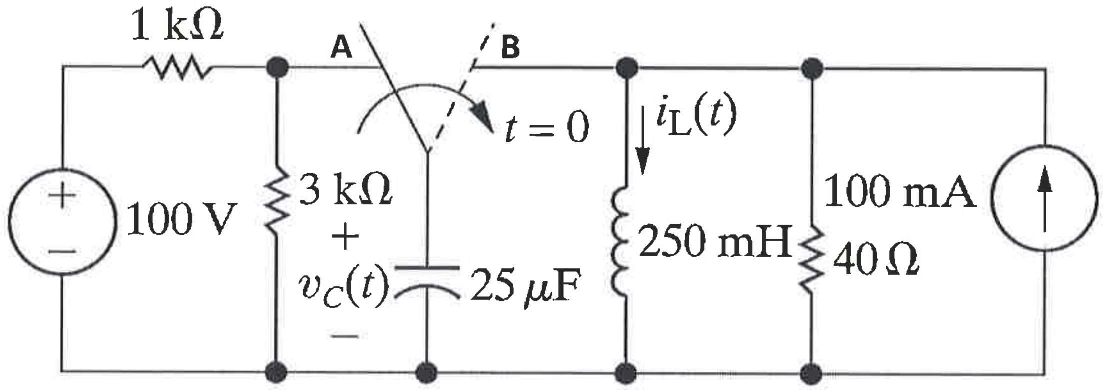
6. For the circuit shown below, the switch has been in position **A** for a long time before switching to position **B** at t=0 (40 points).



a. Draw a sketch of the state of the circuit just before the switch changes to position **B** (e.g. $t=0^-$). Find $i_L(0^-)$ and $v_c(0^-)$.

b. Draw a sketch of the state of the circuit just after the switch changes to position **B** (e.g. $t=0^+$). Find $i_L(0^+)$ and $v_c(0^+)$.

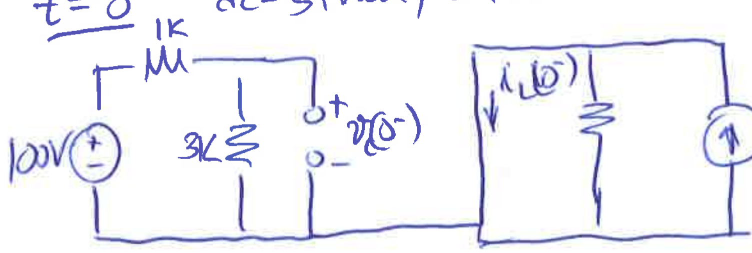
c. Draw a sketch of the state of the circuit after the switch has been in position **B** for a long time ($t \to \infty$). Find $i_L(\infty)$ and $v_c(\infty)$.

d. Calculate the resonant radian, ω_o, and the neper frequency, α, for the circuit.

e. Determine if the transient response of the circuit is overdamped, critically damped, or underdamped.

f. Write the expression for the inductor current, $i_L(t)$ for $t \geq 0$.

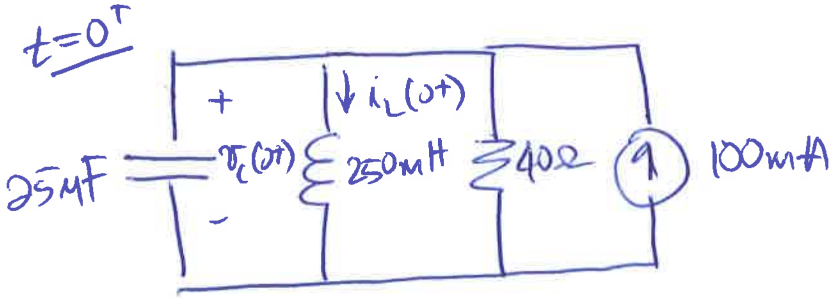
(a) $t=0^-$ "dc-steady state"



$v_c(0^-) = \frac{3}{4}(100) = 75V$

$i_L(0^-) = 100mA = 0.1A$

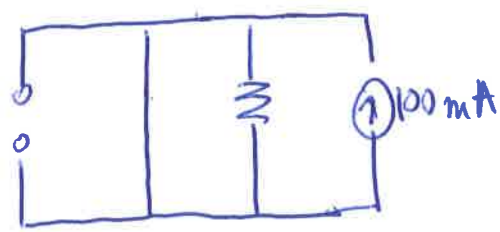
(b) $t=0^+$



$v_c(0^+) = v_c(0^-) = 75V$

$i_L(0^+) = i_L(0^-) = 100mA = 0.1A$

(c) $t \to \infty$ dc steady state



$v_c(\infty) = 0$

$i_L(\infty) = 100mA = 0.1A$

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(d) $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(40)(25\times10^{-6})} = 500$ rad/sec.

$$W_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(250\times10^{-3})(25\times10^{-6})}} = \frac{10^4}{25} = 400 \text{ rad/sec}$$

(e) $\alpha > W_0 \Rightarrow$ overdamped

(f) $i_L(t) = I_f + A_1 e^{+s_1 t} + A_2 e^{+s_2 t}, \quad t \geq 0$

$$S_{1,2} = -500 \pm \sqrt{(500)^2 - (400)^2}$$
$$= -500 \pm \sqrt{25\times10^4 - 16\times10^4} = -500 \pm \sqrt{9\times10^4}$$
$$= -500 \pm 300$$

$s_1 = -200, \quad S_2 = -800$

$I_f = i_L(\infty) = 0.1\,A = 100\,mA$

$i_L(0) = I_f + A_1 + A_2 = 0.1$

$$A_1 + A_2 = 0 \quad \text{--- (1)}$$

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$$\frac{di_L(0^+)}{dt} = +s_1 A_1 + s_2 A_2 = \frac{v_L(0^+)}{L} = \frac{v_c(0^+)}{L} = \frac{75}{250\times10^{-3}}$$

$$-200\,A_1 - 800\,A_2 = 300\ A/s$$

$$-2A_1 - 8A_2 = 3 \quad \text{--- } (2)$$

from (1) $\Rightarrow$ $A_1 = -A_2$

Substituting in (2) $-2A_1 + 8A_1 = 3$

$$6A_1 = 3$$

$$\boxed{\begin{aligned} A_1 &= 0.5 \\ A_2 &= -0.5 \end{aligned}}$$

$$\boxed{i_L(t) = 0.1 + 0.5\,e^{-200t} - 0.5\,e^{-800t}\ \text{A for } t \geq 0}$$