EE 3321

Electromagnetic Field Theory

Spring 2017

Homework #0

Raymond C. Rumpf
800#

February 30, 2017

Homework is due on Monday, 30-Feb-2017.

Reading

Chapter 3, pp. 151–167. (study)
Chapter 4, pp. 178–200. (study)

Problems

Problem #1

Derive the equation for the volume of a sphere by performing a volume integration in spherical coordinates.

Problem #2

Calculate the gradient of the following function.

$$f(x, y, z) = x^2 y + xyz \ .$$

Problem #1

General equation for volume in spherical coordinates is

$$V = \iiint_V dv = \int_{\theta=?}^{?} \int_{\phi=?}^{?} \int_{r=?}^{?} \sin\theta \, r^2 dr \, d\phi \, d\theta$$

We choose the following limits for integration

$$0 \le r < \infty \qquad 0 \le \theta < \pi \qquad 0 \le \phi < 2\pi$$

A sphere is symmetric so we can limit $0 \le \theta < \pi/2$ if we multiply the integration by 2.

$$V = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \int_{r=0}^{R} \sin\theta \; r^2 dr \, d\phi \, d\theta \quad \times \; 2$$

$$= 2 \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \underbrace{\left(\int_{r=0}^{R} r^2 dr \right)}_{\frac{1}{3}R^3} \sin\theta \, d\phi \, d\theta$$

$$= \frac{2R^3}{3} \int_{\theta=0}^{\pi/2} \underbrace{\left(\int_{\phi=0}^{2\pi} d\phi \right)}_{2\pi} \sin\theta \, d\theta$$

Problem #1 continued

$$V = \frac{4\pi R^3}{3} \int_{\theta=0}^{\pi/2} \sin\theta \, d\theta = \frac{4\pi R^3}{3} \left(-\cos\theta\right)\Big|_0^{\pi/2}$$

$$= \frac{4\pi R^3}{3}\left(-\underbrace{\cos\frac{\pi}{2}}_{0} + \underbrace{\cos 0}_{1}\right) = \frac{4\pi R^3}{3}(0+1)$$

$$\boxed{V = \frac{4\pi R^3}{3}}$$

$R \equiv$ radius of sphere

Problem # 2

Calculate the gradient of $f(x,y,z) = x^2 y + xyz$

$$\nabla f(x,y,z) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(x^2 y + xyz\right) = 2xy + yz = y(2x+z)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^2 y + xyz\right) = x^2 + xz = x(x+z)$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left(x^2 y + xyz\right) = xy$$

$$\boxed{\nabla f(x,y,z) = y(2x+z)\hat{x} + x(x+z)\hat{y} + xy\hat{z}}$$