Reading

Chapter 3, pp. 56-96.

Problems

Problem 1
Identify each function below as either a scalar field or vector field by circling the correct answer.

\[
\begin{align*}
\frac{x^2 + \frac{a}{y} - z}{y} & \quad \text{Scalar Field} & \text{Vector Field} \\
\mathbf{y\hat{a}_x} + 3\mathbf{a_y} & \quad \text{Scalar Field} & \text{Vector Field} \\
2\mathbf{a_r} + \cos \phi \mathbf{a_\phi} & \quad \text{Scalar Field} & \text{Vector Field} \\
\sin \theta \sin \phi & \quad \text{Scalar Field} & \text{Vector Field} \\
3i - \cos \theta j + e^4k & \quad \text{Scalar Field} & \text{Vector Field}
\end{align*}
\]

Problem 2
Derive the equation for the volume of a cone by performing a volume integration in cylindrical coordinates.

Problem 3
Calculate the gradient of the following scalar functions

\[
\begin{align*}
A &= x^3 \frac{1}{y} + ye^z \\
B &= \rho z \cos(\phi) + \ln(z) + \rho^2
\end{align*}
\]

Problem 4
Calculate the divergence of the following vector functions:

\[
\begin{align*}
\vec{A} &= -x^3z\mathbf{\hat{a}_r} + x\mathbf{a_z} \\
\vec{B} &= \frac{\cos(\phi)}{\rho} \mathbf{\hat{a}_\rho} + \rho^3 \mathbf{\hat{a}_\phi} + e^z \sin(\phi) \mathbf{\hat{a}_z}
\end{align*}
\]

Problem 5
Calculate the curl of the following vector functions:

\[
\begin{align*}
\vec{A} &= y^3\mathbf{\hat{a}_x} + 2xz\mathbf{\hat{a}_y} + x^2\mathbf{\hat{a}_z} \\
\vec{B} &= \rho z \cos(\phi) \mathbf{\hat{a}_\rho} + 3\rho^3 \mathbf{\hat{a}_\phi}
\end{align*}
\]

Problem 6
Calculate the Laplacian of the following fields where \( h \) is just a constant:

\[
\begin{align*}
A &= e^{-y} \cos(2x) \sin(hx) & h \equiv \text{constant} \\
\vec{B} &= yz\mathbf{\hat{a}_x} + 3xz\mathbf{\hat{a}_z}
\end{align*}
\]