Maxwell’s Equations:  
Gauss’ Law for Magnetic Fields

EE-3321  
Electromagnetic Field Theory

Outline

• No Magnetic Charge  
• Gauss’ Law for Magnetic Fields in Integral Form  
• Gauss’ Law for Magnetic Fields in Differential Form
No Magnetic Charge

Since there exists no isolated magnetic charge, the magnetic field cannot have a beginning or an end. The magnetic field can only form loops.

Prove Zero Magnetic Charge

Surface integral of magnetic flux

Following what we did for electric fields, we calculate total magnetic charge enclosed within a surface $S$ by integrating the flux of the magnetic field lines at the surface.

$$0 = \iiint_S \vec{B} \cdot d\vec{s}$$

No flux lines through surface. $Q_m = 0$

Flux adds to zero. $Q_m = 0$
The result from last slide is Gauss’ law for magnetic fields.

\[ \oint_S \vec{B} \cdot d\vec{s} = 0 \]

Apply Divergence Theorem

The divergence theorem allows us to write a closed-contour surface integral as a volume integral.

\[ \oint_S \vec{A} \cdot d\vec{s} = \iiint_V \left( \nabla \cdot \vec{A} \right) dV \]

Applying this to Gauss’ law for magnetic fields gives us

\[ Q_m = \oint_S \vec{B} \cdot d\vec{s} = \iiint_V \rho_m dV \]

\[ \downarrow \]

\[ Q_m = \iiint_V \left( \nabla \cdot \vec{B} \right) dV = \iiint_V \rho_m dV \]
If the surface $S$ and volume $V$ describe the same space, the argument of both integrals must be equal. Setting these arguments equal gives Gauss' law for magnetic fields in differential form.

$$Q_m = \iiint_V (\nabla \cdot \vec{B}) \, dv = \iiint_V \rho_m \, dv$$

$$\nabla \cdot \vec{B} = \rho_m$$

However, there is no magnetic charge so $\rho_m = 0$.

$$\nabla \cdot \vec{B} = 0$$