Electrostatics: Resistor Examples

EE3321
Electromagnetic Field Theory

Outline

• Parallel plate resistor
• Inhomogeneous resistor
Example #1:
Parallel Plate Resistor

Step 1 – Choose coordinate system.
- **Cartesian**

Step 2 – Assume $V_0$
Example #1

Step 3 – Solve Laplace’s equation

\[ \nabla^2 V = 0 \]

\[ \frac{d^2 V}{dz^2} = 0 \quad V(0) = 0 \text{ and } V(d) = V_0 \]

\[ V(z) = \frac{V_0 z}{d} \quad 0 \leq z \leq d \]

Example #1

Step 4 – Calculate \( \vec{E} \)

\[ \vec{E} = -\nabla V = -\frac{dV}{dz} \hat{a}_z \]

\[ \vec{E} = -\frac{d}{dz} \left( \frac{V_0 z}{d} \right) \hat{a}_z \quad 0 \leq z \leq d \]

\[ \vec{E} = -\frac{V_0}{d} \hat{a}_z \quad 0 \leq z \leq d \]
Example #1

Step 5 – Calculate Current $I$

$$I = \iint_{S} \sigma \mathbf{E} \cdot d\mathbf{s} = \iint_{S} \sigma \left( \frac{V_{0}}{d} \hat{a}_{z} \right) \cdot d\mathbf{s}$$

$$I = -\sigma \frac{V_{0}}{d} \iint_{S} \hat{a}_{z} \cdot d\mathbf{s} = -\frac{\sigma V_{0}}{d} S$$

The sign can be ignored since the direction of the current is known.

$$I = \frac{\sigma V_{0}}{d} S$$

Example #1

Step 6 – Calculate $R$ using $R = \frac{V_{0}}{I}$.

$$R = \frac{V_{0}}{I} = \frac{V_{0}}{\frac{\sigma V_{0}}{d} S} = \frac{d}{\sigma S}$$

$$R = \frac{d}{\sigma S}$$
Example #2: Inhomogeneous Resistor

Suppose we have an inhomogeneous resistor.
Example #2

We split the inhomogeneous resistor into a combination of homogeneous resistors.

Example #2

Calculate each homogeneous resistor independently.

\[ R_i = \frac{d_i}{\sigma_i S_i} = \frac{d_i}{\sigma_i t w_1} \]
Example #2

Calculate each homogeneous resistor independently.

\[
R_1 = \frac{d_1}{\sigma_1 S_1} = \frac{d_1}{\sigma_1 t w_1}
\]

\[
R_2 = \frac{d_1}{\sigma_2 S_2} = \frac{d_1}{\sigma_2 t w_2}
\]

\[
R_3 = \frac{d_2}{\sigma_3 S_3} = \frac{d_2}{\sigma_3 t (w_1 + w_2)}
\]
Example #2

We view our resistor as a series/parallel combination of resistors.

The equivalent resistance is

\[
R_{eq} = R_1 \parallel R_2 + R_3
= \left( \frac{d_1}{\sigma_1 t w_1} \parallel \frac{d_1}{\sigma_2 t w_2} \right) + \frac{d_2}{\sigma_3 (w_1 + w_2)}
\]

\[
R_{eq} = \frac{1}{t} \left[ \frac{d_1 \sigma_1 \sigma_2 w_1 w_2}{\sigma_1 w_1 + \sigma_2 w_2} + \frac{d_2}{\sigma_3 (w_1 + w_2)} \right]
\]