Course Instructor
Dr. Raymond C. Rumpf
Office: A-337
Phone: (915) 747-6958
E-Mail: rcrumpf@utep.edu

EE 4347

Applied Electromagnetics

Topic 4a

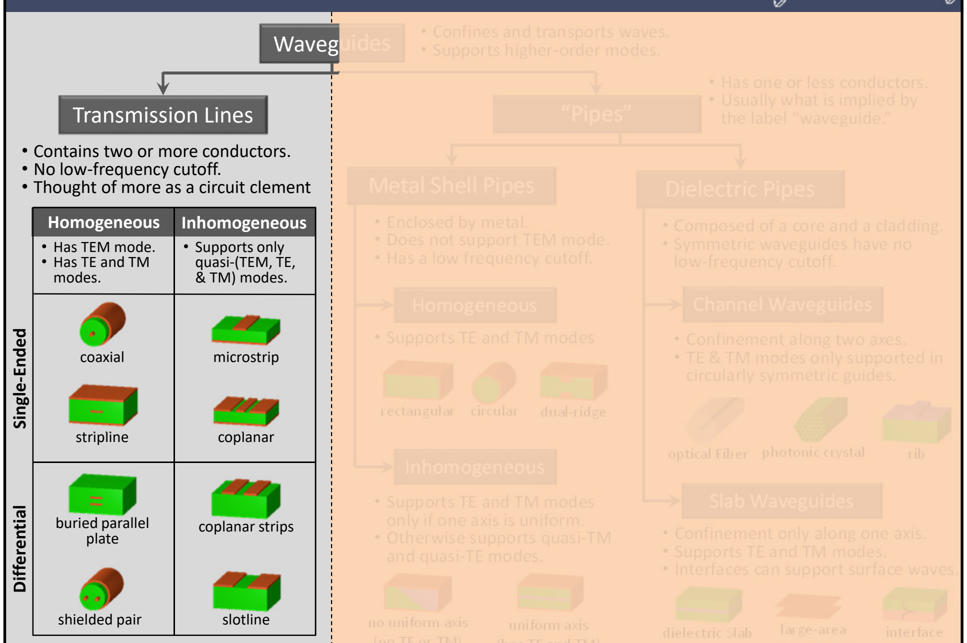
Transmission Lines

Lecture Outline

- Introduction
- Transmission Line Equations
- Transmission Line Wave Equations
- Transmission Line Parameters
  - α and β
  - Characteristic Impedance, Z_0
- Special Cases of Transmission Lines
  - General transmission lines
  - Lossless lines
  - Weakly absorbing lines
  - Distortionless lines
- Examples
  - RG-59 coaxial cable
  - Microstrip design

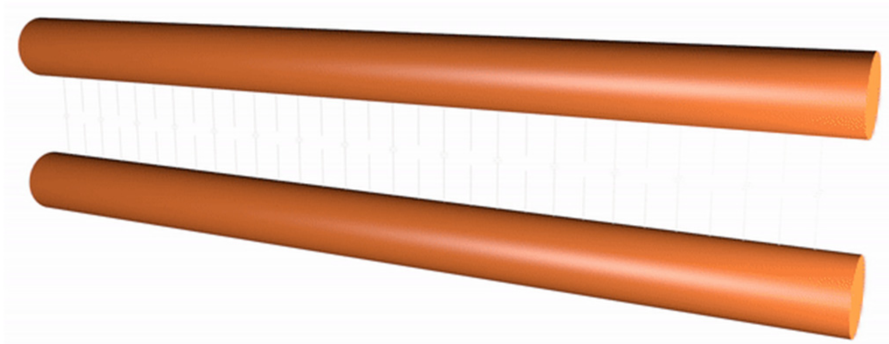
Introduction

Map of Waveguides (LI Media)



Transmission Line Parameters RLGC

We can think transmission lines as being composed of millions of tiny little circuit elements that are distributed along the length of the line.

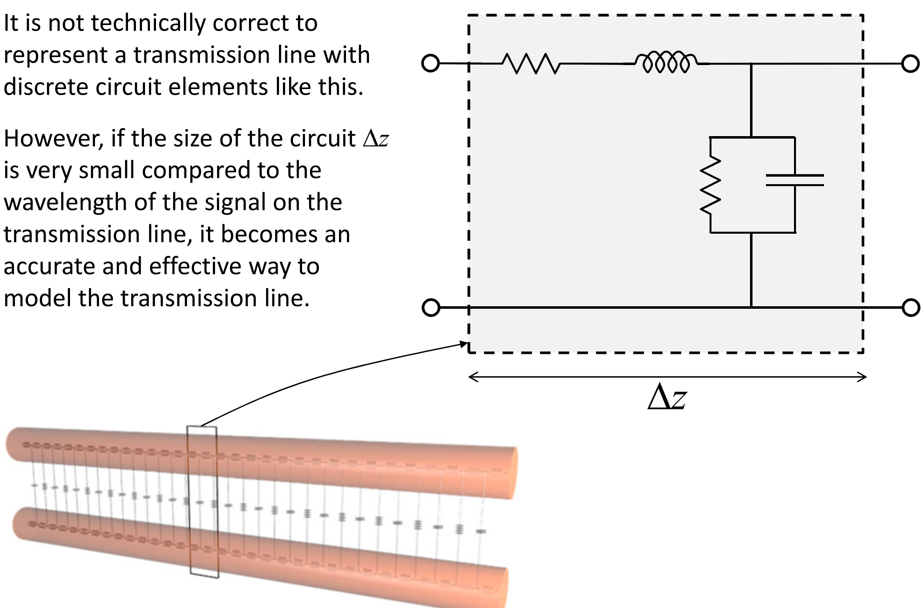


In fact, these circuit element are not discrete, but continuous along the length of the transmission line.

RLGC Circuit Model

It is not technically correct to represent a transmission line with discrete circuit elements like this.

However, if the size of the circuit Δz is very small compared to the wavelength of the signal on the transmission line, it becomes an accurate and effective way to model the transmission line.



L-Type Equivalent Circuit Model

Distributed Circuit Parameters

R (Ω/m)
Resistance per unit length.
Arises due to resistivity in the conductors.

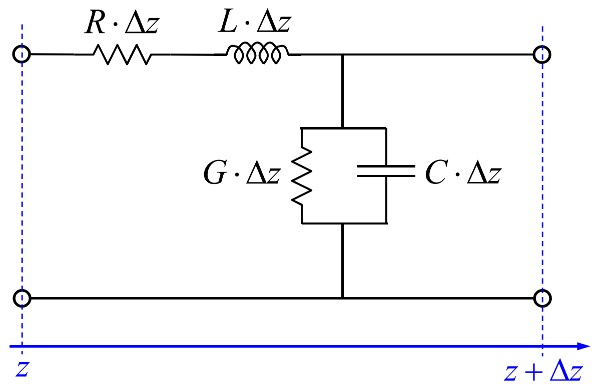
L (H/m)
Inductance per unit length.
Arises due to stored magnetic energy around the line.

G ($1/\Omega\cdot$m)
Conductance per unit length.
Arises due to conductivity in the dielectric separating the conductors.

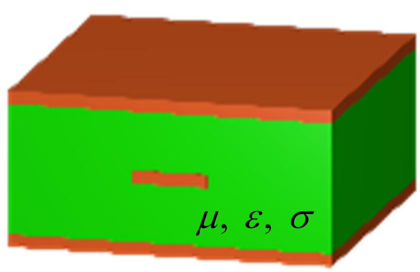
$$G \neq \frac{1}{R}$$

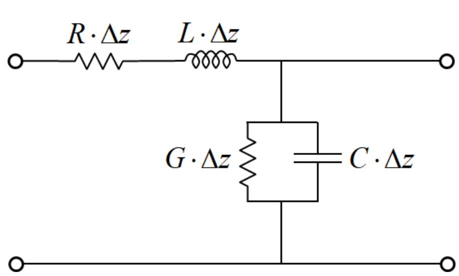
C (F/m)
Capacitance per unit length.
Arises due to stored electric energy between the conductors.

There are many possible circuit models for transmission lines, but most produce the same equations after analysis.



Relation to Electromagnetic Parameters





Every transmission line with a homogeneous fill has:

$$LC = \mu\varepsilon$$

$$\frac{G}{C} = \frac{\sigma}{\varepsilon}$$

Fundamental Vs. Intuitive Parameters

Fundamental Parameters	Intuitive Parameters
Electromagnetics $\mu,\ \varepsilon,\ \sigma$	Electromagnetics $n,\ \eta,\ \alpha,\ \beta,\ \tan\delta$
Transmission Lines $R,\ L,\ G,\ C$	Transmission Lines $Z_0,\ \alpha,\ \beta,\ \text{VSWR}$

The fundamental parameters are the most basic parameters needed to solve a transmission line problem.

However, it is difficult to be intuitive about how they affect signals on the line.

An electromagnetic analysis is needed to determine R, L, G, and C from the geometry of the transmission line.

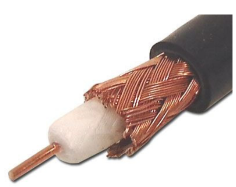
The intuitive parameters provide intuitive insight about how signals behave on a transmission line.

They isolate specific information to a single parameter.

The intuitive parameters are calculated from R, L, G, and C .

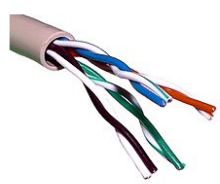
Example RLGC Parameters

RG-59 Coax



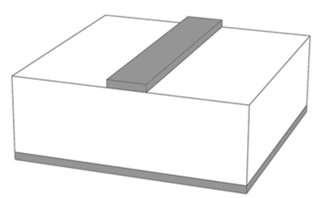
$R = 36\ \text{m}\Omega/\text{m}$
$L = 430\ \text{nH}/\text{m}$
$G = 10\ \mu\mho/\text{m}$
$C = 69\ \text{pF}/\text{m}$
$Z_0 = 75\ \Omega$

CAT5 Twisted Pair



$R = 176\ \text{m}\Omega/\text{m}$
$L = 490\ \text{nH}/\text{m}$
$G = 2\ \mu\mho/\text{m}$
$C = 49\ \text{pF}/\text{m}$
$Z_0 = 100\ \Omega$

Microstrip



$R = 150\ \text{m}\Omega/\text{m}$
$L = 364\ \text{nH}/\text{m}$
$G = 3\ \mu\mho/\text{m}$
$C = 107\ \text{pF}/\text{m}$
$Z_0 = 50\ \Omega$

Surprisingly, almost all transmission lines have parameters very close to these same values.

Transmission Line Equations

E & H → V and I

Fundamentally, all circuit problems are electromagnetic problems and can be solved as such.

All two-conductor transmission lines either support a TEM wave or a wave very closely approximated as TEM.

An important property of TEM waves is that E is uniquely related to V and H and uniquely related to E.

$$V = -\int_L \vec{E} \bullet d\vec{\ell} \qquad\qquad I = \oint_L \vec{H} \bullet d\vec{\ell}$$

This let's us analyze transmission lines in terms of just V and I. This makes analysis much simpler because these are scalar quantities!

Transmission Line Equations

The transmission line equations do for transmission lines the same thing as Maxwell's curl equations do for unguided waves.

Maxwell's Equations

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

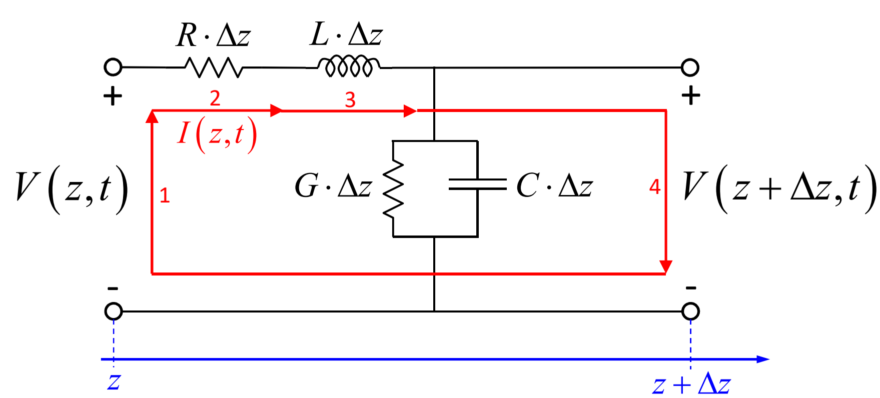
Transmission Line Equations

$$-\frac{\partial V}{\partial z} = RI + L\frac{\partial I}{\partial t}$$

$$-\frac{\partial I}{\partial z} = GV + C\frac{\partial V}{\partial t}$$

Like Maxwell's equations, the transmission line equations are rarely directly useful. Instead, we will derive all of the useful equations from them.

Derivation of First TL Equation (1 of 2)



Apply Kirchoff's voltage law (KVL) to the outer loop of the equivalent circuit:

$$\underbrace{-V(z,t)}_{1} + \underbrace{I(z,t)R\Delta z}_{2} + \underbrace{L\Delta z \frac{\partial I(z,t)}{\partial t}}_{3} + \underbrace{V(z+\Delta z,t)}_{4} = 0$$

Derivation of First TL Equation (2 of 2)

APPLIED EM

We rearrange the equation by bringing all of the voltage terms to the left-hand side of the equation, bringing all of the current terms to the right-hand side of the equation, and then dividing both sides by Δz.

$$-V(z,t)+I(z,t)R\Delta z+L\Delta z\frac{\partial I(z,t)}{\partial t}+V(z+\Delta z,t)=0$$

$$\downarrow$$

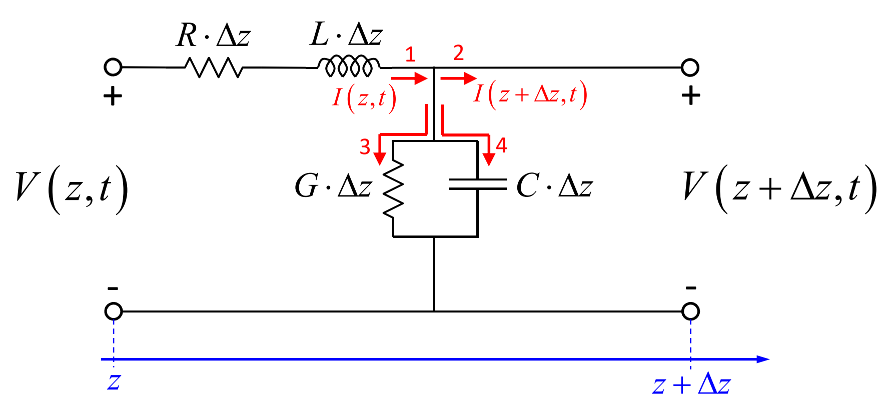
$$-\frac{V(z+\Delta z,t)-V(z,t)}{\Delta z}=RI(z,t)+L\frac{\partial I(z,t)}{\partial t}$$

In the limit as $\Delta z \rightarrow 0$, the expression on the left-hand side becomes a derivative with respect to z.

$$\boxed{-\frac{\partial V(z,t)}{\partial z}=RI(z,t)+L\frac{\partial I(z,t)}{\partial t}}$$

Derivation of Second TL Equation (1 of 2)

APPLIED EM



Apply Kirchoff's current law (KCL) to the main node the equivalent circuit:

$$\underbrace{I(z,t)}_{1}-\underbrace{I(z+\Delta z,t)}_{2}-\underbrace{G\Delta z V(z+\Delta z,t)}_{3}-\underbrace{C\Delta z\frac{\partial V(z+\Delta z,t)}{\partial t}}_{4}=0$$

Derivation of Second TL Equation (2 of 2)

We rearrange the equation by bringing all of the current terms to the left-hand side of the equation, bringing all of the voltage terms to the right-hand side of the equation, and then dividing both sides by Δz.

$$I(z,t) - I(z+\Delta z,t) - G\Delta z V(z+\Delta z,t) - C\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t} = 0$$

$$\downarrow$$

$$-\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = GV(z+\Delta z,t) + C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

In the limit as $\Delta z \rightarrow 0$, the expression on the left-hand side becomes a derivative with respect to z.

$$\boxed{-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}}$$

Transmission Line Wave Equations

Starting Point – Telegrapher Equations

We start with the transmission line equations derived in the previous section.

$$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t} \qquad -\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}$$ time-domain

For time-harmonic (i.e. frequency-domain) analysis, we Fourier transform the equations above.

$$\boxed{-\frac{dV(z)}{dz} = (R + j\omega L)I(z)} \qquad \boxed{-\frac{dI(z)}{dz} = (G + j\omega C)V(z)}$$ frequency-domain

Note: Our derivative d/dz became an ordinary derivative because z is the only independent variable left.

These last equations are commonly referred to as the *telegrapher equations*.

Wave Equation in Terms of $V(z)$

$$-\frac{dV(z)}{dz} = (R + j\omega L)I(z) \quad \text{Eq. (1)} \qquad -\frac{dI(z)}{dz} = (G + j\omega C)V(z) \quad \text{Eq. (2)}$$

To derive a wave equation in terms of $V(z)$, we first differentiate Eq. (1) with respect to z.

$$-\frac{d^2V(z)}{dz^2} = (R + j\omega L)\frac{dI(z)}{dz} \quad \text{Eq. (3)}$$

Second, we substitute Eq. (2) into the right-hand side of Eq. (3) to eliminate $I(z)$ from the equation.

$$-\frac{d^2V(z)}{dz^2} = -(R + j\omega L)(G + j\omega C)V(z)$$

Last, we rearrange the terms to arrive at the final form of the wave equation.

$$\boxed{\frac{d^2V(z)}{dz^2} - (R + j\omega L)(G + j\omega C)V(z) = 0}$$

Wave Equation in Terms of $I(z)$

$$-\frac{dV(z)}{dz}=(R+j\omega L)I(z) \qquad \text{Eq. (1)}$$

$$-\frac{dI(z)}{dz}=(G+j\omega C)V(z) \qquad \text{Eq. (2)}$$

To derive a wave equation in terms of just $I(z)$, we first differentiate Eq. (2) with respect to z.

$$-\frac{d^2I(z)}{dz^2}=(G+j\omega C)\frac{dV(z)}{dz} \qquad \text{Eq. (3)}$$

Second, we substitute Eq. (1) into the right-hand side of Eq. (3) to eliminate $V(z)$ from the equation.

$$-\frac{d^2I(z)}{dz^2}=-(G+j\omega C)(R+j\omega L)I(z)$$

Last, we rearrange the terms to arrive at the final form of the wave equation.

$$\boxed{\frac{d^2I(z)}{dz^2}-(G+j\omega C)(R+j\omega L)I(z)=0}$$

Transmission Lines Slide 21

Propagation Constant, γ

In our wave equations, we have a common term $(G+j\omega C)(R+j\omega L)$.

Define the propagation constant γ to be

$$\gamma=\alpha+j\beta=\sqrt{(G+j\omega C)(R+j\omega L)}$$

Given this definition, the transmission line equations are written as

$$\frac{d^2V(z)}{dz^2}-\gamma^2V(z)=0$$

$$\frac{d^2I(z)}{dz^2}-\gamma^2I(z)=0$$

Transmission Lines Slide 22

Solution to the Wave Equations

If we hand the wave equations off to a mathematician, they will return with the following solutions.

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \quad \rightarrow \quad V(z) = \underbrace{V_0^+ e^{-\gamma z}}_{\text{Forward wave}} + \underbrace{V_0^- e^{\gamma z}}_{\text{Backward wave}}$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \quad \rightarrow \quad I(z) = \underbrace{I_0^+ e^{-\gamma z}}_{\text{Forward wave}} + \underbrace{I_0^- e^{\gamma z}}_{\text{Backward wave}}$$

Both $V(z)$ and $I(z)$ have the same differential equation so it makes sense they have the same solution.

Transmission Lines Slide 23

Transmission Line Parameters:

Attenuation Coefficient, α

Phase Constant, β

Transmission Lines Slide 24

Derivation α and β (1 of 7)

Step 1 – Start with our expression for γ.

$$\gamma = \alpha + j\beta = \sqrt{(G + j\omega C)(R + j\omega L)}$$

Square this expression to get rid of square-root on right-hand side.

$$(\alpha + j\beta)^2 = (G + j\omega C)(R + j\omega L)$$

Expand this expression.

$$\alpha^2 + j2\alpha\beta - \beta^2 = RG + j\omega RC + j\omega LG - \omega^2 LC$$

Collect real and imaginary parts on the left-hand and right-hand sides.

$$(\alpha^2 - \beta^2) + j2\alpha\beta = (RG - \omega^2 LC) + j\omega(RC + LG)$$

Derivation α and β (2 of 7)

Step 2 – Generate two equations by equating real and imaginary parts.

$$2\alpha\beta = \omega(RC + LG)$$

$$(\alpha^2 - \beta^2) + j2\alpha\beta = (RG - \omega^2 LC) + j\omega(RC + LG)$$

$$\alpha^2 - \beta^2 = RG - \omega^2 LC$$

We now have two equations and two unknowns.

Derivation α and β (3 of 7)

Step 3 – Derive a quadratic equation for α^2.

$$2\alpha\beta = \omega\left(RC + LG\right) \qquad \text{Eq. (1a)}$$
$$\alpha^2 - \beta^2 = RG - \omega^2 LC \qquad \text{Eq. (1b)}$$

Solve Eq. (1a) for β.

$$\beta = \frac{\omega}{2\alpha}\left(RC + LG\right) \qquad \text{Eq. (2)}$$

Substitute Eq. (2) into Eq. (1b) and simplify.

$$\alpha^2 - \left[\frac{\omega}{2\alpha}\left(RC + LG\right)\right]^2 = RG - \omega^2 LC$$
$$\alpha^2 - \frac{\omega^2\left(RC + LG\right)^2}{4\alpha^2} = RG - \omega^2 LC$$
$$4\alpha^4 - \omega^2\left(RC + LG\right)^2 = 4\alpha^2 RG - 4\alpha^2\omega^2 LC$$
$$\alpha^4 + \alpha^2\left(\omega^2 LC - RG\right) - \left[\frac{\omega}{2}\left(RC + LG\right)\right]^2 = 0$$

Derivation α and β (4 of 7)

Step 4 – Solve for α^2 using the quadratic formula.

Recall the quadratic formula: $ax^2 + bx + c = 0 \;\rightarrow\; x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Our equation for α is in the form of a quadratic equation where

$$\alpha^4 + \alpha^2\left(\omega^2 LC - RG\right) - \left[\frac{\omega}{2}\left(RC + LG\right)\right]^2 = 0 \;\rightarrow\; \begin{aligned} a &= 1 \\ b &= \omega^2 LC - RG \\ c &= -\left[\frac{\omega}{2}\left(RC + LG\right)\right]^2 \\ x &= \alpha^2 \end{aligned}$$

The solution is

$$\alpha^2 = \frac{-\left(\omega^2 LC - RG\right) \pm \sqrt{\left(\omega^2 LC - RG\right)^2 + 4\left[\frac{\omega}{2}\left(RC + LG\right)\right]}}{2}$$
$$= \frac{RG - \omega^2 LC \pm \sqrt{\left(R^2 + \omega^2 L^2\right)\left(G^2 + \omega^2 C^2\right)}}{2}$$

Derivation α and β (5 of 7)

Step 5 – Resolve the sign of the square-root.

$$\alpha^2 = \frac{RG - \omega^2 LC \pm \sqrt{\left(R^2 + \omega^2 L^2\right)\left(G^2 + \omega^2 C^2\right)}}{2}$$

In order for this expression to always give a real value for α, the sign of the square-root must be positive.

The final expression is

$$\alpha^2 = \frac{RG - \omega^2 LC + \sqrt{\left(R^2 + \omega^2 L^2\right)\left(G^2 + \omega^2 C^2\right)}}{2}$$

Derivation α and β (6 of 7)

Step 6 – Solve for β^2 using our expression for α^2.

Recall Eq. (1b): $\alpha^2 - \beta^2 = RG - \omega^2 LC$

$$\alpha^2 = \frac{RG - \omega^2 LC + \sqrt{\left(R^2 + \omega^2 L^2\right)\left(G^2 + \omega^2 C^2\right)}}{2}$$

We obtain an equation for β^2 by substituting our expression for α^2 into Eq. (1b).

$$\frac{RG - \omega^2 LC + \sqrt{\left(R^2 + \omega^2 L^2\right)\left(G^2 + \omega^2 C^2\right)}}{2} - \beta^2 = RG - \omega^2 LC$$

$$\downarrow$$

$$\beta^2 = -\frac{RG - \omega^2 LC - \sqrt{\left(R^2 + \omega^2 L^2\right)\left(G^2 + \omega^2 C^2\right)}}{2}$$

Derivation α and β (7 of 7)

Step 7 – We arrive at our final expressions for α and β in terms of the fundamental parameters R, L, G, and C by taking the square-root of our latest expressions for α^2 and β^2.

$$\alpha = \pm\sqrt{\frac{\left(RG-\omega^2 LC\right)+\sqrt{\left(R^2+\omega^2 L^2\right)\left(G^2+\omega^2 C^2\right)}}{2}}$$

$$\beta = \pm\sqrt{\frac{-\left(RG-\omega^2 LC\right)+\sqrt{\left(R^2+\omega^2 L^2\right)\left(G^2+\omega^2 C^2\right)}}{2}}$$

Both α and β must be positive quantities for passive materials. This means we take the positive sign for the square-root.

$$\boxed{\alpha = \sqrt{\frac{\left(RG-\omega^2 LC\right)+\sqrt{\left(R^2+\omega^2 L^2\right)\left(G^2+\omega^2 C^2\right)}}{2}}}$$

$$\boxed{\beta = \sqrt{\frac{-\left(RG-\omega^2 LC\right)+\sqrt{\left(R^2+\omega^2 L^2\right)\left(G^2+\omega^2 C^2\right)}}{2}}}$$

Transmission Line Parameters:

Characteristic Impedance, Z_0

Characteristic Impedance, Z_0 (Ω)

The characteristic impedance Z_0 of a transmission line is defined as the ratio of the voltage to the current at any point of a forward travelling wave.

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

Definition for a forward travelling wave.

Definition for a backward travelling wave. Notice the negative sign!

Most characteristic impedance values fall in the 50 Ω to 100 Ω range. The specific value of impedance is not usually of importance. What is important is when the impedance changes because this causes reflections, standing waves, and more.

Derivation of Z_0 (1 of 5)

Step 1 – Substitute our solution into the transmission line equations.

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$-\frac{dV(z)}{dz} = (R + j\omega L) I(z) \qquad -\frac{dI(z)}{dz} = (G + j\omega C) V(z)$$

$$\Downarrow$$

$$-\frac{d}{dz}\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) = (R + j\omega L)\left(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right)$$

$$-\frac{d}{dz}\left(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right) = (G + j\omega C)\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right)$$

Derivation of Z_0 (2 of 5)

Step 2 – Expand the equations and calculate the derivatives.

$$-\frac{d}{dz}\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) = \left(R + j\omega L\right)\left(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right)$$

$$\Downarrow$$

$$\gamma V_0^+ e^{-\gamma z} - \gamma V_0^- e^{\gamma z} = \left(R + j\omega L\right) I_0^+ e^{-\gamma z} + \left(R + j\omega L\right) I_0^- e^{\gamma z}$$

$$-\frac{d}{dz}\left(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right) = \left(G + j\omega C\right)\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right)$$

$$\Downarrow$$

$$\gamma I_0^+ e^{-\gamma z} - \gamma I_0^- e^{\gamma z} = \left(G + j\omega C\right) V_0^+ e^{-\gamma z} + \left(G + j\omega C\right) V_0^- e^{\gamma z}$$

Derivation of Z_0 (3 of 5)

Step 3 – Equate the expressions multiplying the common exponential terms.

$$\gamma V_0^+ = \left(R + j\omega L\right) I_0^+$$

$$\underbrace{\gamma V_0^+}\, e^{-\gamma z} \overbrace{-\gamma V_0^-}\, e^{\gamma z} = \underbrace{\left(R + j\omega L\right) I_0^+}\, e^{-\gamma z} + \overbrace{\left(R + j\omega L\right) I_0^-}\, e^{\gamma z}$$

$$-\gamma V_0^- = \left(R + j\omega L\right) I_0^-$$

$$\gamma I_0^+ = \left(G + j\omega C\right) V_0^+$$

$$\underbrace{\gamma I_0^+}\, e^{-\gamma z} \overbrace{-\gamma I_0^-}\, e^{\gamma z} = \underbrace{\left(G + j\omega C\right) V_0^+}\, e^{-\gamma z} + \overbrace{\left(G + j\omega C\right) V_0^-}\, e^{\gamma z}$$

$$-\gamma I_0^- = \left(G + j\omega C\right) V_0^-$$

Derivation of Z_0 (4 of 5)

Step 4 – Solve each of our four equations for V_0/I_0 to derive expressions for Z_0.

$$\gamma V_0^+ = (R + j\omega L) I_0^+ \quad \rightarrow \quad \frac{V_0^+}{I_0^+} = \frac{R + j\omega L}{\gamma} = Z_0$$

$$-\gamma V_0^- = (R + j\omega L) I_0^- \quad \rightarrow \quad -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma} = Z_0$$

$$\gamma I_0^+ = (G + j\omega C) V_0^+ \quad \rightarrow \quad \frac{V_0^+}{I_0^+} = \frac{\gamma}{G + j\omega C} = Z_0$$

$$-\gamma I_0^- = (G + j\omega C) V_0^- \quad \rightarrow \quad -\frac{V_0^-}{I_0^-} = \frac{\gamma}{G + j\omega C} = Z_0$$

Derivation of Z_0 (5 of 5)

Step 5 – Put Z_0 in terms of just R, L, G, and C.

Recall our expression for γ: $\gamma = \alpha + j\beta = \sqrt{(G + j\omega C)(R + j\omega L)}$

We can substitute this into either of our expressions for Z_0.

$$Z_0 = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C}$$

Proceed with the first expression.

$$Z_0 = \frac{R + j\omega L}{\sqrt{(G + j\omega C)(R + j\omega L)}} = \sqrt{\frac{(R + j\omega L)^2}{(G + j\omega C)(R + j\omega L)}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Final Expression for Z_0 (Ω)

We have derived a general expression for the characteristic impedance Z_0 of a transmission line in terms of the fundamental parameters R, L, G, and C.

Definition: $$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

Expression: $$Z_0 = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Transmission Lines Slide 39

Dissecting the Characteristic Impedance, Z_0

The characteristic impedance describes the amplitude and phase relation between voltage and current along a transmission line. With this picture in mind, the characteristic impedance can be written as

$$Z_0 = |Z_0| \measuredangle \theta_{Z_0}$$

$$V(z) = V_0^+ e^{-\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} = \frac{V_0^+}{Z_0} e^{-\gamma z} = |Z_0| V_0^+ e^{-\gamma z} e^{j\measuredangle\theta_{Z_0}}$$

The characteristic impedance can also be written in terms of its real and imaginary parts.

$$Z_0 = R_0 + jX_0$$

Reactive part of Z_0. This is not equal to $j\omega L$ or $1/j\omega C$.

Resistive part of Z_0. This is not equal to R or G.

Transmission Lines Slide 40

Special Cases of Transmission Lines:

General Transmission Line

Parameters for General TLs

Propagation Constant, γ

$$\gamma = \alpha + j\beta = \sqrt{(G + j\omega C)(R + j\omega L)}$$

Attenuation Coefficient, α

$$\alpha = \sqrt{\frac{(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

Phase Constant, β

$$\beta = \sqrt{\frac{-(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

Characteristic Impedance, Z_0

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Special Cases of Transmission Lines:

Lossless Lines

Definition of Lossless TL

When we think about transmission lines, we tend to think of the special case of the lossless line because the equations simplify considerably.

For a transmission line to be lossless, it must have

$$R = G = 0$$

Parameters for Lossless TLs

Propagation Constant, γ

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

Attenuation Coefficient, α

$$\alpha = 0$$

Phase Constant, β

$$\beta = \omega\sqrt{LC}$$

Characteristic Impedance, Z_0

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{L}{C}}$$

$$R_0 = \sqrt{\frac{L}{C}} \qquad X_0 = 0$$

Special Cases of Transmission Lines:

Weakly Absorbing Line

Definition of Weakly Absorbing TL

Most practical transmission lines have loss, but very low loss making them weakly absorbing.

We will define a weakly absorbing line as

$$R \leq \omega L \quad \text{and} \quad G \ll \omega C$$

Ensures very little conduction between the lines through the dielectric.

Ensures low ohmic loss for signals propagating through the line.

Parameters for Weakly Absorbing TLs

Attenuation Coefficient, α

$$\alpha \approx \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)$$

Special Cases of Transmission Lines:

Distortionless Lines

Definition of Distortionless TL

APPLIED EM

In a real transmission line, different frequencies will be attenuated differently because α is a function of ω. This causes distortion in the signals carried by the line.

$$\alpha(\omega) = \sqrt{\frac{\left(RG - \omega^2 LC\right) + \sqrt{\left(R^2 + \omega^2 L^2\right)\left(G^2 + \omega^2 C^2\right)}}{2}}$$

To be distortionless, there must be a choice of R, L, G, and C that eliminates ω from the expression of α, effectively making α independent of frequency ω.

The necessary condition to be distortionless is

$$\frac{R}{L} = \frac{G}{C}$$

Parameters for Distortionless TLs

Propagation Constant, γ

$$\gamma = \alpha + j\beta = \sqrt{RG} + j\omega\sqrt{LC}$$

Attenuation Coefficient, α

$$\alpha = \sqrt{RG}$$

Phase Constant, β

$$\beta = \omega\sqrt{LC}$$

To be distortionless, we must have $\beta \propto \omega$. β is a measure of how quickly a signal accumulates phase. Different frequencies have different wavelengths and therefore must accumulate different phase through the same length of line.

Characteristic Impedance, Z_0

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

$$R_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \qquad X_0 = 0$$

Example:

Properties of RG-59 Coax

The Lossless Circular Coax

Fundamental Parameters (derived in EE 3321)

$$L = \frac{\mu}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right] \quad (\mathrm{H/m})$$

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} \quad (\mathrm{F/m})$$

Attenuation Coefficient, α

$$\alpha = 0$$

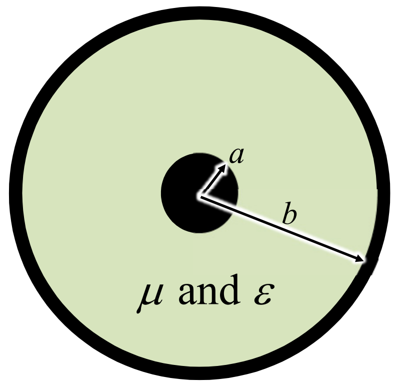
Phase Constant, β

$$\beta = \omega\sqrt{\mu\varepsilon}$$

Characteristic Impedance, Z_0

$$Z_0 = R_0 + jX_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right) \qquad a \gg b$$

$$R_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right) \qquad X_0 = 0$$



Typical RLGC for RG-59 Coax at 2 GHz

The typical RG-59 coaxial cable operating at 2.0 GHz has the following RLGC parameters:

$$R = 36 \ \mathrm{m\Omega/m}$$
$$L = 430 \ \mathrm{nH/m}$$
$$G = 10 \ \mu\mho/\mathrm{m}$$
$$C = 69 \ \mathrm{pF/m}$$

Calculate the transmission line parameters γ, α, β, and Z_0.

Classify the line as lossless, weakly absorbing, distortionless, etc.

Solution (1 of 3)

Our equations mostly utilize the angular frequency ω instead of the ordinary frequency f.

$$\omega = 2\pi f = 2\pi\left(2.0\times10^{9}\ \text{s}^{-1}\right) = \underline{12.5664\times10^{9}\ \text{rad/s}}$$

The characteristic impedance Z_0 is

$$\begin{aligned} Z_0 &= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \\ &= \sqrt{\frac{(36\ \text{m}\Omega/\text{m}) + j(12.5664\times10^{9}\ \text{rad/s})(430\ \text{nH/m})}{(10\ \mu\mho/\text{m}) + j(12.5664\times10^{9}\ \text{rad/s})(69\ \text{pF/m})}} \\ &= \boxed{78.94 + j1.92\times10^{-4}\ \Omega} \end{aligned}$$

Note the imaginary part of Z_0 is very small indicating that our line is very low loss.

Solution (2 of 3)

The complex propagation constant γ is

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{\begin{aligned} &\left[(36\ \text{m}\Omega/\text{m}) + j(12.5664\times10^{9}\ \text{rad/s})(430\ \text{nH/m})\right] \\ &\cdot\left[(10\ \mu\mho/\text{m}) + j(12.5664\times10^{9}\ \text{rad/s})(69\ \text{pF/m})\right] \end{aligned}} \\ &= \boxed{6.23\times10^{-4} + j68.45\ \text{m}^{-1}} \end{aligned}$$

From this result, we read off α and β.

$$\gamma = \alpha + j\beta = 6.23\times10^{-4} + j68.45\ \text{m}^{-1}$$

$$\boxed{\alpha = 6.23\times10^{-4}\ \text{Np/m}}$$ Np is Nepers

$$\boxed{\beta = 68.45\ \text{rad/m}}$$ rad is radians

Solution (3 of 3)

Is the line lossless? → NO

No because $R \neq 0$ and $G \neq 0$.
Also, we can determine this because $\alpha \neq 0$.

Is the line weakly absorbing? → YES

$$R \stackrel{?}{\leq} \omega L$$
$$(36\ \text{m}\Omega/\text{m}) \stackrel{?}{\leq} (12.5664\times10^{9}\ \text{rad/s})(430\ \text{nH/m})$$
$$0.036 \stackrel{?}{\leq} 5403.5$$
$$\text{Yes}$$

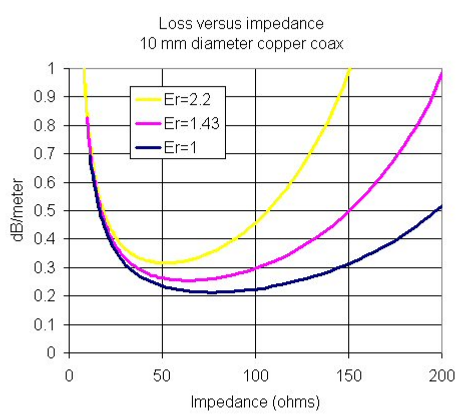
$$G \stackrel{?}{\ll} \omega C$$
$$(10\ \mu\mho/\text{m}) \stackrel{?}{\leq} (12.5664\times10^{9}\ \text{rad/s})(69\ \text{pF/m})$$
$$10\times10^{-6} \stackrel{?}{\leq} 0.8671$$
$$\text{Yes}$$

Is the line distortionless? → NO, but close

$$RC \stackrel{?}{=} LG$$
$$(36\ \text{m}\Omega/\text{m})(69\ \text{pF/m}) \stackrel{?}{\leq} (430\ \text{nH/m})(10\ \mu\mho/\text{m})$$
$$2.48\times10^{-12} \stackrel{?}{\leq} 4.30\times10^{-12}$$
$$\text{No, but close}$$

Cable Loss Vs. Characteristic Impedance

As we adjust the cable dimensions (i.e. b/a), we change both its impedance and its loss characteristics. This let's us plot the cable loss vs. characteristic impedance for a coax with different dielectric fills.



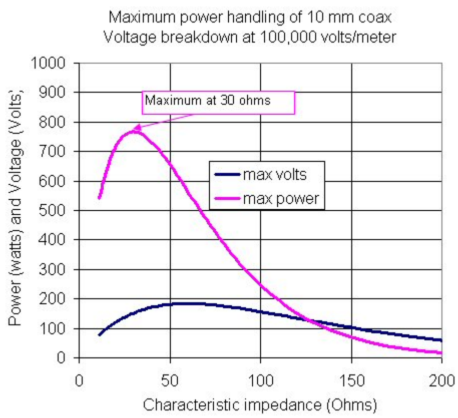
For the air-filled coax, we observe minimum loss at around 77 Ω, where $b/a \approx 3.5$.

A coaxial cable filled with polyethelene (ε_r = 2.2), the minimum loss occurs at 51.2 Ω (b/a = 3.6).

https://www.microwaves101.com/encyclopedias/why-fifty-ohms

Power Handling Vs. Characteristic Impedance

As we adjust the cable dimensions (i.e. b/a), we affect the peak voltage handling capability (breakdown) and its power handling capability (heat).



We observe the lowest peak voltage at just over 50 Ω which we interpret as the point of best voltage handling capability.

We observe the lowest peak current at around 30 Ω which we interpret as the point of best power handling capability.

https://www.microwaves101.com/encyclopedias/why-fifty-ohms

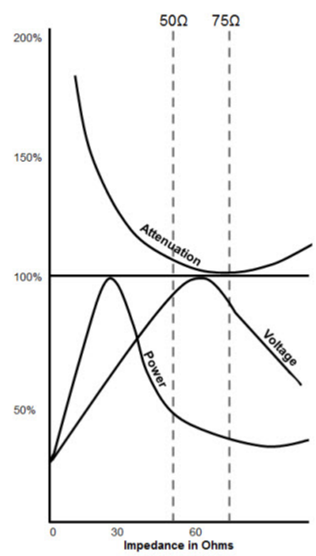
Why 50 Ω Impedance is Best?

Two researchers, Lloyd Espenscheid and Herman Affel, working at Bell Labs produced this graph in 1929. They needed to send 4 MHz signals hundreds. Transmission lines capable of handling high voltage and high power were needed in order to accomplish this.

Data to the right was generated for an air-filled coaxial cable.

Best for High Voltage: Z_0 = 60 Ω
Best for High Power: Z_0 = 30 Ω
Best for Attenuation: Z_0 = 75 Ω

50 Ω seems like the best compromise.



Why 75 Ω Impedance Standard for Coax?

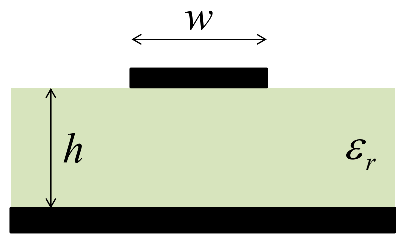
Nobody really knows!!

The ideal impedance is closer to 50 Ω, however this requires a thicker center conductor. Maybe 75 Ω is a compromise between low loss and mechanical flexibility?

Example:

Microstrip Design

The Lossless Microstrip



Attenuation Coefficient, α

$$\alpha = 0$$

Phase Constant, β

$$\varepsilon_{r,\text{eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2\sqrt{1 + 12\,h/w}}$$

$$\beta \approx k_0 \sqrt{\varepsilon_{r,\text{eff}}}$$

Characteristic Impedance, Z_0

$$Z_0 = R_0 + jX_0 \cong \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{\text{eff}}}} \ln\left(\dfrac{8h}{w} + \dfrac{w}{4h}\right) & w/h \leq 1 \quad \text{thin lines} \\ \dfrac{1}{\sqrt{\varepsilon_{\text{eff}}}} \dfrac{120\pi}{\left[w/h + 1.393 + 0.667 \ln\left(w/h + 1.444\right)\right]} & w/h > 1 \quad \text{wide lines} \end{cases}$$

Transmission Lines — Slide 63

Problem Description

Typically, the manufacturing process fixes the value of dielectric constant ε_r. This means the impedance of microstrips is controlled solely through the ratio w/h.

For this example, design a 50 Ω microstrip transmission line in FR-4, which as a dielectric constant of 4.5, to operate at 2.4 GHz.

$$\frac{w}{h} = ?$$

Transmission Lines — Slide 64

Design Equations

To solve this problem, we must first derive some design equations. To do this, we solve our microstrip equations for w/h. This gives

$$A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

$$B = \frac{60\pi^2}{Z_0\sqrt{\varepsilon_r}}$$

$$\frac{w}{h} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2} & w/h \le 2 \quad \text{thin lines} \\[2ex] \dfrac{2}{\pi}\left\{B - 1 - \ln(2B - 1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B - 1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right]\right\} & w/h > 2 \quad \text{wide lines} \end{cases}$$

Design Solution (1 of 2)

Applying our design equations, we get

$$A = 1.5438$$

$$B = 5.5831$$

$$\frac{w}{h} = \begin{cases} 1.8799 & w/h \le 2 \quad \text{thin lines} \\ 1.8812 & w/h > 2 \quad \text{wide lines} \end{cases}$$

Since the above numbers for w/h are essentially the same, we conclude that

$$\frac{w}{h} \cong 1.88$$

Design Solution (2 of 2)

We learn from our manufacturing engineer that a convenient choice for substrate thickness h is 0.5 mm. From this, to get 50 Ω the width w of the microstrip should be

$$w \cong 1.88h = 1.88\left(0.5\text{ mm}\right) = \boxed{0.94\text{ mm}}$$

The phase constant for this line will be

$$\varepsilon_{\text{eff}} = 3.3941$$

$$k_0 = \frac{\omega}{c_0} = \frac{2\pi f}{c_0} = \frac{2\pi\left(2.4\times10^9\text{ s}^{-1}\right)}{299792458\text{ m/s}} = 50.3\text{ m}^{-1}$$

$$\beta \cong \left(50.3\text{ m}^{-1}\right)\sqrt{3.3941} = \boxed{92.67\text{ m}^{-1}}$$