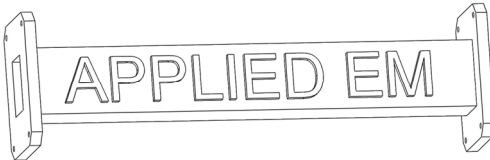


**Course Instructor**  
 Dr. Raymond C. Rumpf  
 Office: A-337  
 Phone: (915) 747-6958  
 E-Mail: rcrumpf@utep.edu




**EE 4347**  
**Applied Electromagnetics**

**Topic 4b**

# Transmission Line Parameters

◆◆ These notes may contain copyrighted material obtained under fair use rules. Distribution of these materials is strictly prohibited ◆◆

1

**Lecture Outline**


- Attenuation Coefficient  $\alpha$  and Phase Constant  $\beta$
- Characteristic Impedance,  $Z_0$
- Special Cases of Transmission Lines
  - General transmission lines
  - Lossless lines
  - Weakly absorbing lines
  - Distortionless lines
- Why is  $50 \Omega$  a Standard Impedance?

Transmission Line Parameters
Slide 2

2

## Fundamental Vs. Intuitive Parameters APPLIED EM

Fundamental Parameters	Intuitive Parameters
Electromagnetics $\mu, \epsilon, \sigma$	Electromagnetics $n, \eta, \alpha, \beta, \tan \delta$
Transmission Lines $R, L, G, C$	Transmission Lines $Z_0, \alpha, \beta, \text{VSWR}$

The fundamental parameters are the most basic parameters needed to solve a transmission line problem.

However, it is difficult to be intuitive about how they affect signals on the line.

An electromagnetic analysis is needed to determine  $R, L, G,$  and  $C$  from the geometry of the transmission line.

The intuitive parameters provide intuitive insight about how signals behave on a transmission line.

They isolate specific information to a single parameter.

The intuitive parameters are calculated from  $R, L, G,$  and  $C$ .

Transmission Line Parameters
Slide 3

3

# Attenuation Coefficient $\alpha$ and Phase Constant $\beta$

Transmission Line Parameters
Slide 4

4

## Derivation $\alpha$ and $\beta$ (1 of 7)



Step 1 – Start with the expression for  $\gamma$ .

$$\gamma = \alpha + j\beta = \sqrt{(G + j\omega C)(R + j\omega L)}$$

Square this expression to get rid of square-root on right-hand side.

$$(\alpha + j\beta)^2 = (G + j\omega C)(R + j\omega L)$$

Expand this expression.

$$\alpha^2 + j2\alpha\beta - \beta^2 = RG + j\omega RC + j\omega LG - \omega^2 LC$$

Collect real and imaginary parts on the left-hand and right-hand sides.

$$(\alpha^2 - \beta^2) + j2\alpha\beta = (RG - \omega^2 LC) + j\omega(RC + LG)$$

Transmission Line Parameters



Slide 5

5

## Derivation $\alpha$ and $\beta$ (2 of 7)



Step 2 – Generate two equations by equating real and imaginary parts.

$$\begin{array}{c}
 2\alpha\beta = \omega(RC + LG) \\
 \swarrow \quad \searrow \\
 (\alpha^2 - \beta^2) + j2\alpha\beta = (RG - \omega^2 LC) + j\omega(RC + LG) \\
 \swarrow \quad \searrow \\
 \alpha^2 - \beta^2 = RG - \omega^2 LC
 \end{array}$$

There are now two equations and two unknowns.

$$2\alpha\beta = \omega(RC + LG)$$

$$\alpha^2 - \beta^2 = RG - \omega^2 LC$$

Transmission Line Parameters



Slide 6

6

## Derivation $\alpha$ and $\beta$ (3 of 7)



Step 3 – Derive a quadratic equation for  $\alpha^2$ .

$$2\alpha\beta = \omega(RC + LG) \quad \text{Eq. (1a)}$$

$$\alpha^2 - \beta^2 = RG - \omega^2 LC \quad \text{Eq. (1b)}$$

Solve Eq. (1a) for  $\beta$ .

$$\beta = \frac{\omega}{2\alpha}(RC + LG) \quad \text{Eq. (2)}$$

Substitute Eq. (2) into Eq. (1b) and simplify.

$$\alpha^2 - \left[ \frac{\omega}{2\alpha}(RC + LG) \right]^2 = RG - \omega^2 LC$$

$$\alpha^2 - \frac{\omega^2(RC + LG)^2}{4\alpha^2} = RG - \omega^2 LC$$

$$4\alpha^4 - \omega^2(RC + LG)^2 = 4\alpha^2 RG - 4\alpha^2 \omega^2 LC$$

$$\alpha^4 + \alpha^2(\omega^2 LC - RG) - \left[ \frac{\omega}{2}(RC + LG) \right]^2 = 0$$

Transmission Line Parameters



Slide 7

7

## Derivation $\alpha$ and $\beta$ (4 of 7)



Step 4 – Solve for  $\alpha^2$  using the quadratic equation.

Recall the quadratic formula:  $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The equation for  $\alpha$  is in the form of the quadratic equation where

$$a = 1$$

$$b = \omega^2 LC - RG$$

$$\alpha^4 + \alpha^2(\omega^2 LC - RG) - \left[ \frac{\omega}{2}(RC + LG) \right]^2 = 0 \rightarrow c = -\left[ \frac{\omega}{2}(RC + LG) \right]^2$$

$$x = \alpha^2$$

The solution is

$$\alpha^2 = \frac{-(\omega^2 LC - RG) \pm \sqrt{(\omega^2 LC - RG)^2 + 4 \left[ \frac{\omega}{2}(RC + LG) \right]^2}}{2}$$

$$= \frac{RG - \omega^2 LC \pm \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}$$

Transmission Line Parameters



Slide 8

8

## Derivation $\alpha$ and $\beta$ (5 of 7)



Step 5 – Resolve the sign of the square-root.

$$\alpha^2 = \frac{RG - \omega^2 LC \pm \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}$$

In order for this expression to always give a real value for  $\alpha$ , the sign of the square-root must be positive.

The final expression is

$$\alpha^2 = \frac{RG - \omega^2 LC + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}$$

Transmission Line Parameters



Slide 9

9

## Derivation $\alpha$ and $\beta$ (6 of 7)



Step 6 – Solve for  $\beta^2$  using the expression for  $\alpha^2$ .

Recall Eq. (1b):  $\alpha^2 - \beta^2 = RG - \omega^2 LC$

$$\alpha^2 = \frac{RG - \omega^2 LC + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}$$

Derive an equation for  $\beta^2$  by substituting the expression for  $\alpha^2$  into Eq. (1b).

$$\frac{RG - \omega^2 LC + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2} - \beta^2 = RG - \omega^2 LC$$

↓

$$\beta^2 = -\frac{RG - \omega^2 LC - \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}$$

Transmission Line Parameters



Slide 10

10

## Derivation $\alpha$ and $\beta$ (7 of 7)



Step 7 – The final expressions for  $\alpha$  and  $\beta$  are derived in terms of the fundamental parameters  $R$ ,  $L$ ,  $G$ , and  $C$  by taking the square-root of the latest expressions for  $\alpha^2$  and  $\beta^2$ .

$$\alpha = \pm \sqrt{\frac{(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

$$\beta = \pm \sqrt{\frac{-(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

Both  $\alpha$  and  $\beta$  must be positive quantities for passive materials.  
This means the positive sign is taken for the square-roots.

$$\alpha = \sqrt{\frac{(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

$$\beta = \sqrt{\frac{-(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

Transmission Line Parameters



Slide 11

11

# Characteristic Impedance $Z_0$

Transmission Line Parameters



Slide 12

12

## Characteristic Impedance, $Z_0$ ( $\Omega$ )



The characteristic impedance  $Z_0$  of a transmission line is defined as the ratio of the voltage to the current at any point of a forward travelling wave.

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

Definition for a forward travelling wave.

Definition for a backward travelling wave. Notice the negative sign!

Most characteristic impedance values fall in the  $50 \Omega$  to  $100 \Omega$  range. The specific value of impedance is not usually of importance. What is important is when the impedance changes because this causes reflections, standing waves, and more.

Transmission Line Parameters



Slide 13

13

## Derivation of $Z_0$ (1 of 5)



Step 1 – Substitute the solutions back into the transmission line equations.

$$\begin{array}{ccc}
 V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} & & \\
 I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} & & \\
 \downarrow & & \downarrow \\
 -\frac{dV(z)}{dz} = (R + j\omega L)I(z) & & -\frac{dI(z)}{dz} = (G + j\omega C)V(z) \\
 \downarrow & & \downarrow \\
 -\frac{d}{dz}(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}) & & -\frac{d}{dz}(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}) \\
 = (R + j\omega L)(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}) & & = (G + j\omega C)(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z})
 \end{array}$$

Transmission Line Parameters



Slide 14

14

## Derivation of $Z_0$ (2 of 5)



Step 2 – Expand the equations and calculate the derivatives.

$$-\frac{d}{dz}(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}) \\ = (R + j\omega L)(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z})$$



$$\gamma V_0^+ e^{-\gamma z} - \gamma V_0^- e^{\gamma z} \\ = (R + j\omega L)I_0^+ e^{-\gamma z} + (R + j\omega L)I_0^- e^{\gamma z}$$

$$-\frac{d}{dz}(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}) \\ = (G + j\omega C)(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z})$$



$$\gamma I_0^+ e^{-\gamma z} - \gamma I_0^- e^{\gamma z} \\ = (G + j\omega C)V_0^+ e^{-\gamma z} + (G + j\omega C)V_0^- e^{\gamma z}$$

Transmission Line Parameters



Slide 15

15

## Derivation of $Z_0$ (3 of 5)



Step 3 – Equate the expressions multiplying the common exponential terms.

$$\gamma V_0^+ = (R + j\omega L)I_0^+ \\ \gamma V_0^+ e^{-\gamma z} - \gamma V_0^- e^{\gamma z} = (R + j\omega L)I_0^+ e^{-\gamma z} + (R + j\omega L)I_0^- e^{\gamma z} \\ -\gamma V_0^- = (R + j\omega L)I_0^-$$

$$\gamma I_0^+ = (G + j\omega C)V_0^+ \\ \gamma I_0^+ e^{-\gamma z} - \gamma I_0^- e^{\gamma z} = (G + j\omega C)V_0^+ e^{-\gamma z} + (G + j\omega C)V_0^- e^{\gamma z} \\ -\gamma I_0^- = (G + j\omega C)V_0^-$$

Transmission Line Parameters



Slide 16

16



## Derivation of $Z_0$ (4 of 5)



Step 4 – Solve each of our four equations for  $V_0/I_0$  to derive expressions for  $Z_0$ .

$$\gamma V_0^+ = (R + j\omega L) I_0^+ \rightarrow \frac{V_0^+}{I_0^+} = \frac{R + j\omega L}{\gamma} = Z_0$$

$$-\gamma V_0^- = (R + j\omega L) I_0^- \rightarrow -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma} = Z_0$$

$$\gamma I_0^+ = (G + j\omega C) V_0^+ \rightarrow \frac{V_0^+}{I_0^+} = \frac{\gamma}{G + j\omega C} = Z_0$$

$$-\gamma I_0^- = (G + j\omega C) V_0^- \rightarrow -\frac{V_0^-}{I_0^-} = \frac{\gamma}{G + j\omega C} = Z_0$$

Transmission Line Parameters



Slide 17

17

## Derivation of $Z_0$ (5 of 5)



Step 5 – Put  $Z_0$  in terms of just  $R$ ,  $L$ ,  $G$ , and  $C$ .

Recall our expression for  $\gamma$ :  $\gamma = \alpha + j\beta = \sqrt{(G + j\omega C)(R + j\omega L)}$

We can substitute this into either of our expressions for  $Z_0$ .

$$Z_0 = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C}$$

Proceed with the first expression.

$$\begin{aligned} Z_0 &= \frac{R + j\omega L}{\gamma} = \frac{R + j\omega L}{\sqrt{(G + j\omega C)(R + j\omega L)}} \\ &= \frac{(R + j\omega L)^2}{\sqrt{(G + j\omega C)(R + j\omega L)}} \\ &= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \end{aligned}$$

Transmission Line Parameters



Slide 18

18

## Final Expression for $Z_0$ ( $\Omega$ )



We have derived a general expression for the characteristic impedance  $Z_0$  of a transmission line in terms of the fundamental parameters  $R$ ,  $L$ ,  $G$ , and  $C$ .

Definition: 
$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

Expressions: 
$$Z_0 = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Transmission Line Parameters



Slide 19

19

## Dissecting the Characteristic Impedance, $Z_0$



The characteristic impedance describes the amplitude and phase relation between voltage and current along a transmission line. With this picture in mind, the characteristic impedance can be written as

$$Z_0 = |Z_0| \angle \theta_{Z_0}$$

$$V(z) = V_0^+ e^{-\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} = \frac{V_0^+}{Z_0} e^{-\gamma z} = \frac{V_0^+}{|Z_0|} e^{-\gamma z} e^{j\angle \theta_{Z_0}}$$

The characteristic impedance can also be written in terms of its real and imaginary parts.

$$Z_0 = R_0 + jX_0$$

Reactive part of  $Z_0$ . This is not equal to  $j\omega L$  or  $1/j\omega C$ .

Resistive part of  $Z_0$ . This is not equal to  $R$  or  $G$ .

Transmission Line Parameters



Slide 20

20

# Special Cases of Transmission Lines:

## General Transmission Line

Transmission Line Parameters



Slide 21

21

### Parameters for General TLs



Propagation Constant,  $\gamma$

$$\gamma = \alpha + j\beta = \sqrt{(G + j\omega C)(R + j\omega L)}$$

Attenuation Coefficient,  $\alpha$

$$\alpha = \sqrt{\frac{(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

Phase Constant,  $\beta$

$$\beta = \sqrt{\frac{-(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

Characteristic Impedance,  $Z_0$

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Transmission Line Parameters



Slide 22

22

# Special Cases of Transmission Lines:

## Lossless Lines

Transmission Line Parameters



Slide 23

23

### Definition of Lossless TL



When we think about transmission lines, we tend to think of the special case of the lossless line because the equations simplify considerably.

For a transmission line to be lossless, it must have

$$R = G = 0$$

Transmission Line Parameters



Slide 24

24

## Parameters for Lossless TLs



Propagation Constant,  $\gamma$

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

Attenuation Coefficient,  $\alpha$

$$\alpha = 0$$

Phase Constant,  $\beta$

$$\beta = \omega\sqrt{LC}$$

Characteristic Impedance,  $Z_0$

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{L}{C}}$$

$$R_0 = \sqrt{\frac{L}{C}} \quad X_0 = 0$$

Transmission Line Parameters



Slide 25

25

## Special Cases of Transmission Lines:

### Weakly Absorbing Line

Transmission Line Parameters



Slide 26

26

## Definition of Weakly Absorbing TL



Most practical transmission lines have loss, but very low loss making them weakly absorbing.

We will define a weakly absorbing line as

$$R \leq \omega L \quad \text{and} \quad G \ll \omega C$$

Ensures low ohmic loss for signals propagating through the line.

Ensures very little conduction between the lines through the dielectric.

Transmission Line Parameters



Slide 27

27

## Parameters for Weakly Absorbing Tls



Attenuation Coefficient,  $\alpha$

$$\alpha \approx \frac{1}{2} \left( \frac{R}{Z_0} + GZ_0 \right)$$

Conductance through the dielectric dominates attenuation in high-impedance transmission lines.

Resistivity in the conductors dominates attenuation in low-impedance transmission lines.

In weakly absorbing transmission lines, there usually exists a "sweet spot" for the impedance where attenuation is minimized.

Transmission Line Parameters



Slide 28

28

# Special Cases of Transmission Lines: Distortionless Lines

Transmission Line Parameters



Slide 29

29

## Definition of Distortionless TL



In a real transmission line, different frequencies will be attenuated differently because  $\alpha$  is a function of  $\omega$ . This causes distortion in the signals carried by the line.

$$\alpha(\omega) = \sqrt{\frac{(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

To be distortionless, there must be a choice of  $R$ ,  $L$ ,  $G$ , and  $C$  that eliminates  $\omega$  from the expression of  $\alpha$ , effectively making  $\alpha$  independent of frequency  $\omega$ .

The necessary condition to be distortionless is

$$\frac{R}{L} = \frac{G}{C} \quad \text{or} \quad RC = LG$$

Transmission Line Parameters



Slide 30

30

## Parameters for Distortionless Tls



Propagation Constant,  $\gamma$

$$\gamma = \alpha + j\beta = \sqrt{RG} + j\omega\sqrt{LC}$$

Attenuation Coefficient,  $\alpha$

$$\alpha = \sqrt{RG}$$

Phase Constant,  $\beta$

$$\beta = \omega\sqrt{LC}$$

To be distortionless, we must have  $\beta \propto \omega$ .  $\beta$  is a measure of how quickly a signal accumulates phase. Different frequencies have different wavelengths and therefore must accumulate different phase through the same length of line.

Characteristic Impedance,  $Z_0$

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

$$R_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \quad X_0 = 0$$

Transmission Line Parameters



Slide 31

31

# Why 50 $\Omega$ ?

Transmission Line Parameters



Slide 32

32



## Cable Loss Vs. Characteristic Impedance

APPLIED EM

As we adjust the cable dimensions (i.e.  $b/a$ ), we change both its impedance and its loss characteristics. This let's us plot the cable loss vs. characteristic impedance for a coax with different dielectric fills.

Loss versus impedance  
10 mm diameter copper coax

Impedance (ohms)

For the air-filled coax, we observe minimum loss at around  $77 \Omega$ , where  $b/a \approx 3.5$ .

A coaxial cable filled with polyethelene ( $\epsilon_r = 2.2$ ), the minimum loss occurs at  $51.2 \Omega$  ( $b/a = 3.6$ ).

<https://www.microwaves101.com/encyclopedias/why-fifty-ohms>

Transmission Line Parameters
Slide 33

33

## Power Handling Vs. Characteristic Impedance

APPLIED EM

As we adjust the cable dimensions (i.e.  $b/a$ ), we affect the peak voltage handling capability (breakdown) and its power handling capability (heat).

Maximum power handling of 10 mm coax  
Voltage breakdown at 100,000 volts/meter

Characteristic impedance (Ohms)

We observe the lowest peak voltage at just over  $50 \Omega$  which we interpret as the point of best voltage handling capability.

We observe the lowest peak current at around  $30 \Omega$  which we interpret as the point of best power handling capability.

<https://www.microwaves101.com/encyclopedias/why-fifty-ohms>

Transmission Line Parameters
Slide 34

34

## Why 50 $\Omega$ Impedance is Best?

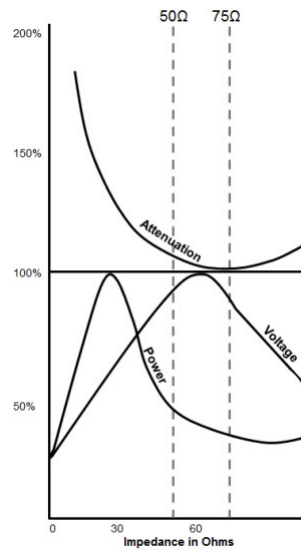


Two researchers, Lloyd Espenscheid and Herman Affel, working at Bell Labs produced this graph in 1929. They needed to send 4 MHz signals hundreds of miles. Transmission lines capable of handling high voltage and high power were needed in order to accomplish this.

The data shown at right was generated for an air-filled coaxial cable.

Best for High Voltage:  $Z_0 = 60 \Omega$   
 Best for High Power:  $Z_0 = 30 \Omega$   
 Best for Low Attenuation:  $Z_0 = 75 \Omega$

50  $\Omega$  seemed like the best compromise.



Transmission Line Parameters



35

## Why 75 $\Omega$ Impedance Standard for Coax?



Nobody really knows!!

The ideal impedance is closer to 50  $\Omega$ , however this requires a thicker center conductor. Maybe 75  $\Omega$  is a compromise between low loss and mechanical flexibility?

Transmission Line Parameters



36