EE 4347
Applied Electromagnetics

Topic 4f

Multi Segment Transmission Line Devices

Lecture Outline

• Quarter-Wave Transformer
• Impedance Matching
• Stubs
• Scattering Parameters
Quarter-Wave Transformer

A quarter-wave transformer is a section of line that is a $\lambda/4$ long.

When the length of the line is $\lambda/4$, then we have

$$\beta \ell = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

This means the signal accumulates 90° of phase. When told a TL is $\lambda/4$, usually no other information is needed.

When this is the case, our impedance transformation equation reduces to

$$Z_{in}(-\ell) = Z_0 \frac{Z_L + jZ_0 \tan(\beta \ell)}{Z_0 + jZ_L \tan(\beta \ell)} = Z_0 \frac{Z_L + jZ_0 \tan(\pi/2)}{Z_0 + jZ_L \tan(\pi/2)}$$

$$= Z_0 \frac{Z_L + jZ_0 \cdot \infty}{Z_0 + jZ_L \cdot \infty} \quad \tan(\pi/2) = \infty.$$
Quarter-Wave Transformer (2 of 2)

Since both the numerator and denominator are $\infty$, we must apply L’Hopital’s rule.

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{f'(x_0)}{g'(x_0)}$$

Applying this to our impedance transformation equation, we get

$$Z_{\text{in}}(-\ell) = \lim_{\beta \to \frac{\pi}{4}} \frac{Z_L + jZ_0 \tan(\beta \ell)}{Z_0 + jZ_L \tan(\beta \ell)}$$

$$= \frac{Z_0}{Z_L}$$

The final equation shows that the load impedance $Z_L$ gets completely inverted. The input impedance becomes the input admittance.

$$Z_{\text{in}}(-\frac{\lambda}{4}) = \frac{Z_0^2}{Z_L}$$
Impedance Inversion (2 of 5)

- Generator
- Quarter-Wave Transmission Line
- Capacitive Load

\[ z = -\lambda/4 \]

- Generator
- Input Impedance is Inductive

\[ \frac{V_g}{Z_g} \quad Z_{in} \quad \gamma, Z_0 \quad C \]

\[ L \]

\[ z = -\lambda/4 \]

- Capacitors look like inductors!

\[ Z_{in} \text{ of Quarter-Wave Line} \]
\[ Z_{in} = \frac{Z_0^2}{Z_L} \]

Equivalent Inductance
\[ L = CZ_0^2 \]

Impedance Inversion (3 of 5)

- Generator
- Quarter-Wave Transmission Line
- Short-Circuit Load

\[ z = -\lambda/4 \]

- Generator
- Input Impedance is an open circuit.

\[ \frac{V_g}{Z_g} \quad Z_{in} \quad \gamma, Z_0 \quad Z_L = 0 \]

\[ z = -\lambda/4 \]

- Short circuits look like open circuits!

\[ Z_{in} \text{ of Quarter-Wave Line} \]
\[ Z_{in} = \frac{Z_0^2}{Z_L} = \infty \]
Impedance Inversion (4 of 5)

Input Impedance is a short circuit.

Input Impedance is a short circuit.

Impedance Inversion (5 of 5)

Input Impedance is $Z_0$.

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Matched loads are always matched!

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Impedance Matching

Similar to the anti-reflection layer for waves, we can match a transmission line to a load impedance by inserting a quarter-wave section of a second transmission line.

\[ Z_{in} = Z_0 \]

\[ \beta_{at} = ? \]

\[ Z_{at} = \sqrt{Z_0 Z_L} \]

\[ z = -\frac{\lambda}{4} \]

\[ z = 0 \]

We must perform an electromagnetic analysis of the transmission line to determine \( \beta_{at} \).

\[ \beta_{at} \approx \omega \sqrt{\mu_0 \varepsilon_r} \]

\[ \ell = \frac{\lambda}{4} = \frac{\pi}{2 \beta_{at}} \]
Example (1 of 3)

A 50 Ω microstrip line on FR-4 ($\varepsilon_r = 4.4$) operates at 2.4 GHz and is connected to patch antenna which has a 120 Ω input impedance. How much power is reflected? How can the circuit be improved?

Reflected Power:

$$\left| \Gamma \right|^2 = \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|^2 = \left( \frac{120 \Omega - (50 \Omega)}{120 \Omega + (50 \Omega)} \right)^2 = 0.4118^2 = 17\%$$

Example (2 of 3)

A 50 Ω microstrip line on FR-4 ($\varepsilon_r = 4.4$) operates at 2.4 GHz and is connected to patch antenna which has a 120 Ω input impedance. How much power is reflected? How can the circuit be improved?

Design:

$$Z_n = \sqrt{Z_L Z_0} = \sqrt{(120 \Omega)(50 \Omega)} = 77.5 \Omega$$

Perform an EM analysis to determine TL dimensions to get 50 Ω. For TEM mode,

$$\beta = \omega \sqrt{\mu \varepsilon} = \frac{2\pi f}{c_0} \sqrt{\mu \varepsilon} = \frac{2\pi (2.4 \times 10^8 \text{ s}^{-1})}{3.0 \times 10^8 \text{ m/s}} \sqrt{(1.0)(4.4)} = 105.44 \text{ rad/s}$$
A 50 Ω microstrip line on FR-4 (εᵣ = 4.4) operates at 2.4 GHz and is connected to patch antenna which has a 120 Ω input impedance. How much power is reflected? How can the circuit be improved?

Design: Given $\beta$, the length of the line should be

$$\beta \ell = \frac{\pi}{2} \implies \ell = \frac{\pi}{2\beta} = \frac{\pi}{2(105.44 \ \text{rad/s})} = 1.4898 \times 10^{-2} \ \text{m}$$

Stubs
What is a Stub? (1 of 3)

What do short circuits look like $\lambda/4$ away?

What is a Stub? (2 of 2)

What do short circuits look like $\lambda/4$ away? Open circuits!
The Shorted Stub is a Band Pass Filter

The circuit is actually shorted for all frequencies other than whatever frequency has wavelength $\lambda$ inside the line. The short circuit blocks all signals.

At the frequency with wavelength $\lambda$, the circuit is not shorted and signals are allowed to pass.

Stubs in Practice

Multi Segment TL Devices
Scattering Parameters

Definition of a Scattering Matrix

The scattering matrix relates the amplitudes of the input waves to the amplitudes of the output waves.

\[
\begin{bmatrix}
V_1^- \\
V_2^- \\
\vdots \\
V_N^-
\end{bmatrix}
= 
\begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1N} \\
S_{21} & S_{22} & \cdots & S_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
S_{N1} & S_{N2} & \cdots & S_{NN}
\end{bmatrix}
\begin{bmatrix}
V_1^+ \\
V_2^+ \\
\vdots \\
V_N^+
\end{bmatrix}
\]

\[S_{ij} = \frac{V_i^-}{V_j^+} \text{ for other applied voltages}\]

Any linear system can be reduced to a single scattering matrix that describes how it behaves.
### S-Matrix for Two-Port Networks

\[
\begin{bmatrix}
\text{Output 1} \\
\text{Output 2}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
\text{Input 1} \\
\text{Input 2}
\end{bmatrix}
\]

- \(S_{11}\) is synonymous with reflection coefficient.
- \(S_{21}\) is synonymous with transmission coefficient.

Very often, engineers will say “\(S\)-1-1” instead of saying “reflection,” and say “\(S\)-2-1” instead of saying transmission.

### Combining S-Matrices

Suppose there are two circuits, A and B, described by scattering matrices that placed in series. What is the scattering matrix of the combined network?

The answer is NOT matrix multiplication!!!

\[
\begin{bmatrix}
S_{11}^{(AB)} & S_{12}^{(AB)} \\
S_{21}^{(AB)} & S_{22}^{(AB)}
\end{bmatrix} \neq
\begin{bmatrix}
S_{11}^{(A)} & S_{12}^{(A)} \\
S_{21}^{(A)} & S_{22}^{(A)}
\end{bmatrix}\begin{bmatrix}
S_{11}^{(B)} & S_{12}^{(B)} \\
S_{21}^{(B)} & S_{22}^{(B)}
\end{bmatrix}
\]

Instead, it is a Redheffer star product.

\[
\begin{bmatrix}
S_{11}^{(AB)} & S_{12}^{(AB)} \\
S_{21}^{(AB)} & S_{22}^{(AB)}
\end{bmatrix} =
\begin{bmatrix}
S_{11}^{(A)} & S_{12}^{(A)} \\
S_{21}^{(A)} & S_{22}^{(A)}
\end{bmatrix} \otimes
\begin{bmatrix}
S_{11}^{(B)} & S_{12}^{(B)} \\
S_{21}^{(B)} & S_{22}^{(B)}
\end{bmatrix}
\]
Redheffer Star Product

\[
\begin{bmatrix}
S_{11}^{(AB)} & S_{12}^{(AB)} \\
S_{21}^{(AB)} & S_{22}^{(AB)}
\end{bmatrix}
= \begin{bmatrix}
S_{11}^{(A)} & S_{12}^{(A)} \\
S_{21}^{(A)} & S_{22}^{(A)}
\end{bmatrix}
\otimes \begin{bmatrix}
S_{11}^{(B)} & S_{12}^{(B)} \\
S_{21}^{(B)} & S_{22}^{(B)}
\end{bmatrix}
\]

\[
S_{11}^{(AB)} = \frac{S_{11}^{(A)} - S_{22}^{(A)} S_{11}^{(B)} S_{21}^{(B)} + S_{12}^{(A)} S_{21}^{(B)} S_{11}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}
\]

\[
S_{22}^{(AB)} = \frac{S_{22}^{(B)} - S_{11}^{(B)} S_{22}^{(A)} S_{12}^{(A)} + S_{21}^{(B)} S_{12}^{(A)} S_{22}^{(A)}}{1 - S_{22}^{(B)} S_{11}^{(B)}}
\]

\[
S_{12}^{(AB)} = \frac{S_{12}^{(A)} S_{12}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}
\]

\[
S_{21}^{(AB)} = \frac{S_{21}^{(A)} S_{21}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}
\]