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EE 4347

Applied Electromagnetics

Topic 4f

Multi Segment Transmission Line Devices

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Lecture Outline

- Quarter-Wave Transformer
- Impedance Matching
- Stubs
- Scattering Parameters

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Quarter-Wave Transformer

Quarter-Wave Transformer (1 of 2)

A quarter-wave transformer is a section of line that is a $\lambda/4$ long.

When the length of the line is $\lambda/4$, then we have

$$\beta\ell = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}$$

This means the signal accumulates 90° of phase.
When told a TL is $\lambda/4$, usually no other information is needed.

When this is the case, our impedance transformation equation reduces to

$$Z_{\text{in}}\left(-\ell\right) = Z_0\frac{Z_L + jZ_0\tan\left(\beta\ell\right)}{Z_0 + jZ_L\tan\left(\beta\ell\right)} = Z_0\frac{Z_\text{L} + jZ_0\tan\left(\pi/2\right)}{Z_0 + jZ_\text{L}\tan\left(\pi/2\right)}$$

$$= Z_0\frac{Z_\text{L} + jZ_0\cdot\infty}{Z_0 + jZ_\text{L}\cdot\infty}$$

$\tan(\pi/2) = \infty$.

$$= ?$$

Quarter-Wave Transformer (2 of 2)

APPLIED EM

Since both the numerator and denominator are ∞, we must apply L'Hopital's rule.

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{f'(x_0)}{g'(x_0)}$$

Applying this to our impedance transformation equation, we get

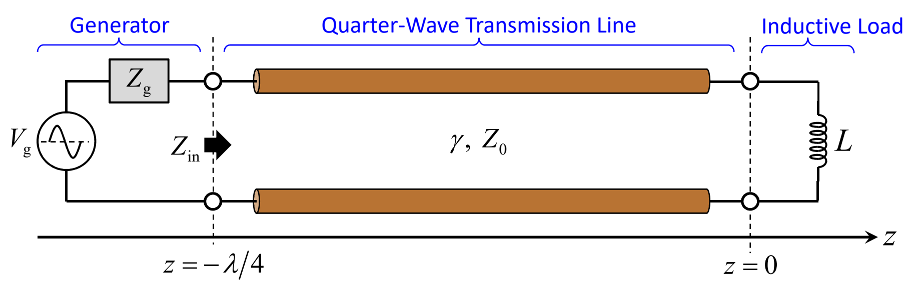
$$Z_{\text{in}}(-\ell) = \lim_{\beta\ell \to \pi/2} Z_0 \frac{Z_{\text{L}} + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_{\text{L}} \tan(\beta\ell)} = Z_0 \frac{jZ_0 \sec^2(\beta\ell)}{jZ_{\text{L}} \sec^2(\beta\ell)} = \frac{Z_0^2}{Z_{\text{L}}}$$

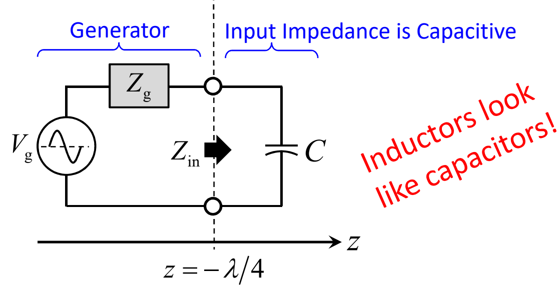
The final equation shows that the load impedance Z_{L} gets completely inverted. The input impedance becomes the input admittance.

$$\boxed{Z_{\text{in}}\left(-\frac{\lambda}{4}\right) = \frac{Z_0^2}{Z_{\text{L}}}}$$

Impedance Inversion (1 of 5)

APPLIED EM





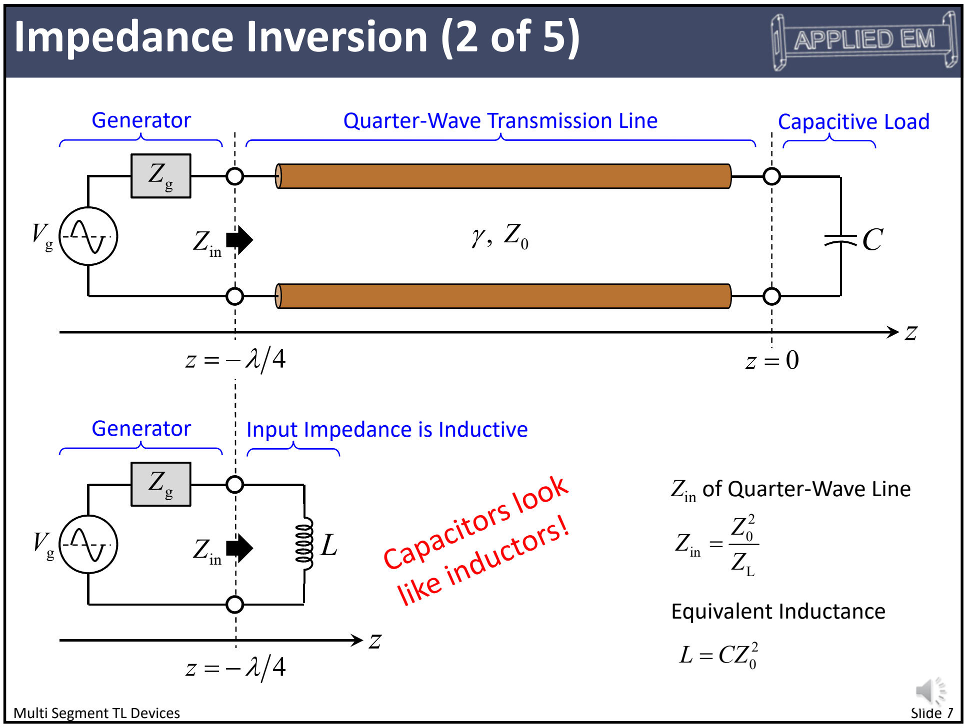
Inductors look like capacitors!

Z_{in} of Quarter-Wave Line

$$Z_{\text{in}} = \frac{Z_0^2}{Z_{\text{L}}}$$

Equivalent Capacitance

$$C = \frac{L}{Z_0^2}$$


Impedance Inversion (2 of 5)
APPLIED EM
Generator
Quarter-Wave Transmission Line
Capacitive Load
Z_g
V_g
Z_{in}
γ, Z_0
C
z
$z = -\lambda/4$
$z = 0$
Generator
Input Impedance is Inductive
L
Capacitors look like inductors!
Z_{in} of Quarter-Wave Line
$Z_{in} = \frac{Z_0^2}{Z_L}$
Equivalent Inductance
$L = CZ_0^2$
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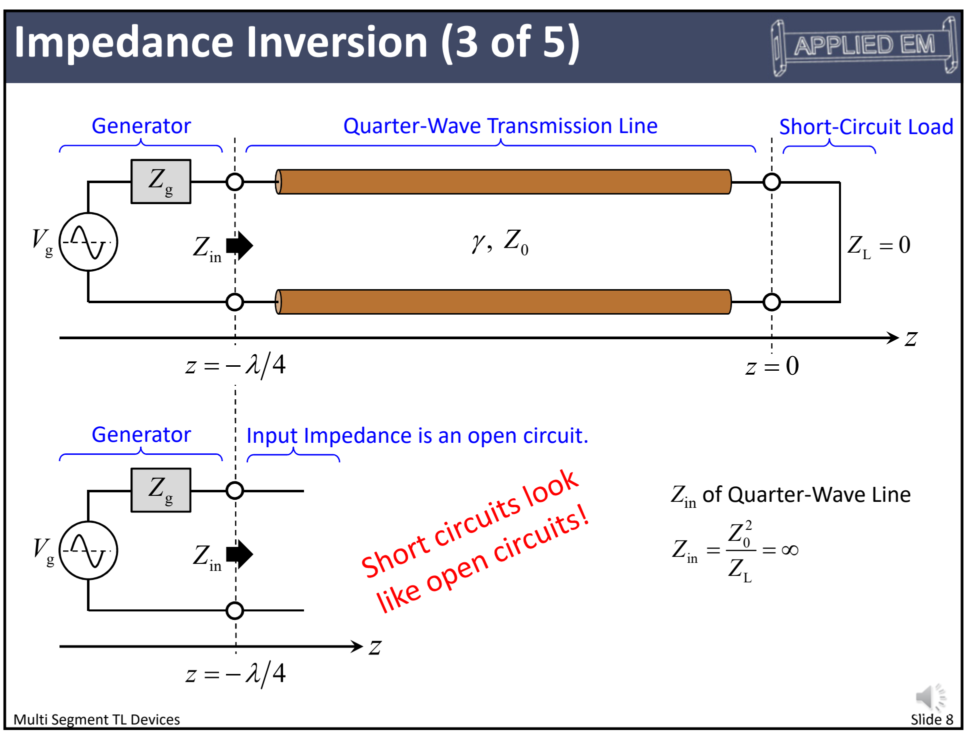
Impedance Inversion (3 of 5)
APPLIED EM
Generator
Quarter-Wave Transmission Line
Short-Circuit Load
Z_g
V_g
Z_{in}
γ, Z_0
$Z_L = 0$
z
$z = -\lambda/4$
$z = 0$
Generator
Input Impedance is an open circuit.
Short circuits look like open circuits!
Z_{in} of Quarter-Wave Line
$Z_{in} = \frac{Z_0^2}{Z_L} = \infty$
Multi Segment TL Devices
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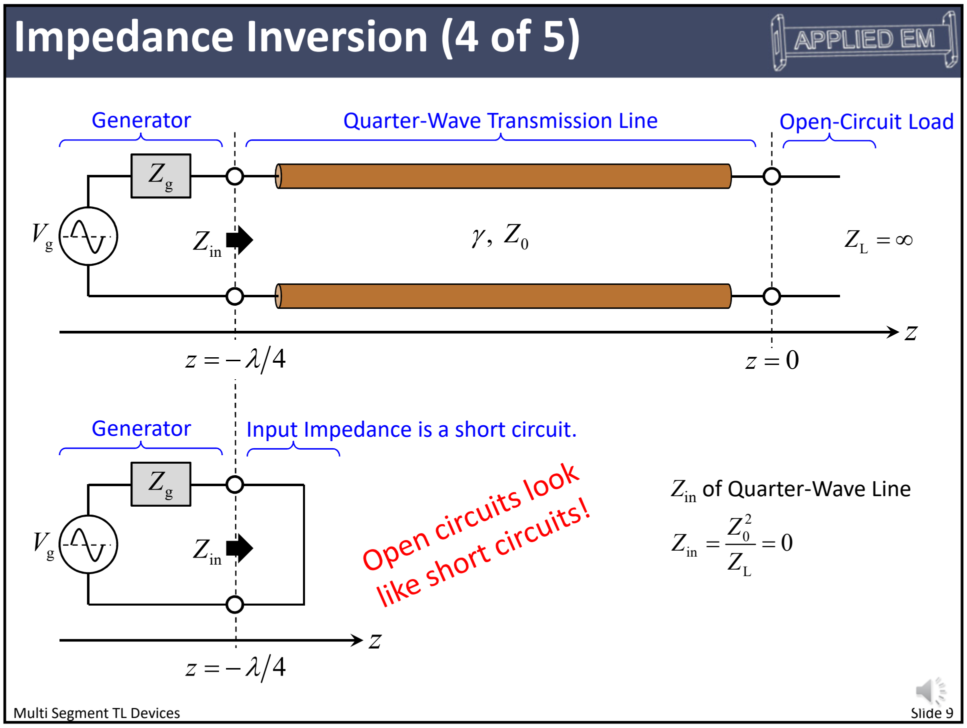
Impedance Inversion (4 of 5)
APPLIED EM
Generator
Quarter-Wave Transmission Line
Open-Circuit Load
Z_g
V_g
Z_{in}
$\gamma,\ Z_0$
$Z_L = \infty$
$z = -\lambda/4$
$z = 0$
z
Generator
Input Impedance is a short circuit.
Open circuits look like short circuits!
Z_{in} of Quarter-Wave Line
$Z_{in} = \frac{Z_0^2}{Z_L} = 0$
Multi Segment TL Devices
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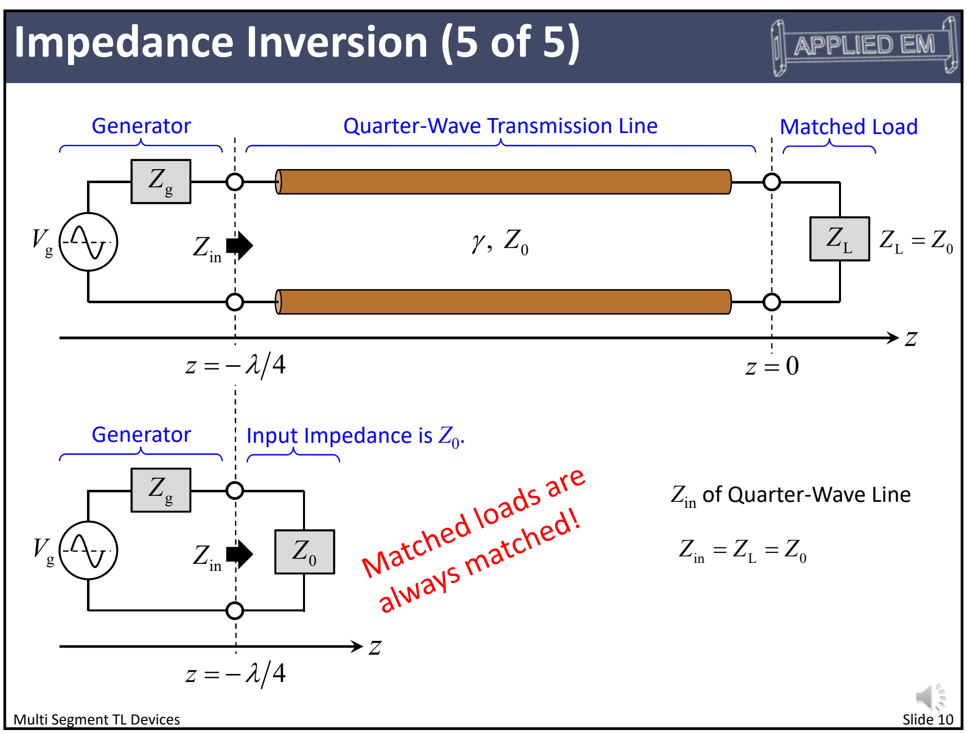
Impedance Inversion (5 of 5)
APPLIED EM
Generator
Quarter-Wave Transmission Line
Matched Load
Z_g
V_g
Z_{in}
$\gamma,\ Z_0$
Z_L
$Z_L = Z_0$
$z = -\lambda/4$
$z = 0$
z
Generator
Input Impedance is Z_0.
Z_0
Matched loads are always matched!
Z_{in} of Quarter-Wave Line
$Z_{in} = Z_L = Z_0$
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Impedance Matching

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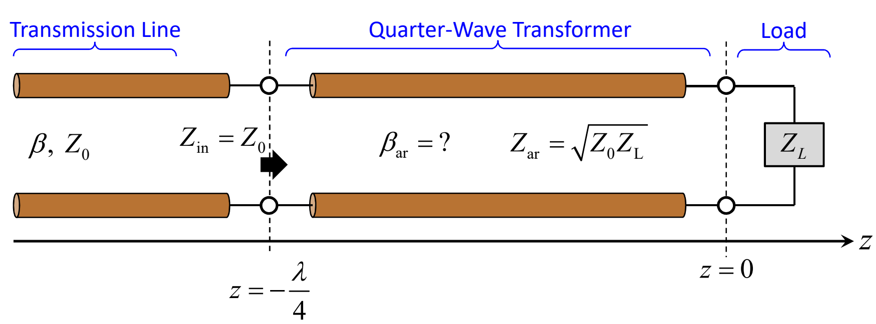
Impedance Matching

APPLIED EM

Similar to the anti-reflection layer for waves, we can match a transmission line to a load impedance by inserting a quarter-wave section of a second transmission line.



We must perform an electromagnetic analysis of the transmission line to determine β_{ar}.

$$\beta_{ar} \approx \omega\sqrt{\mu_r \varepsilon_r} \qquad \ell = \frac{\lambda}{4} = \frac{\pi}{2\beta_{ar}}$$

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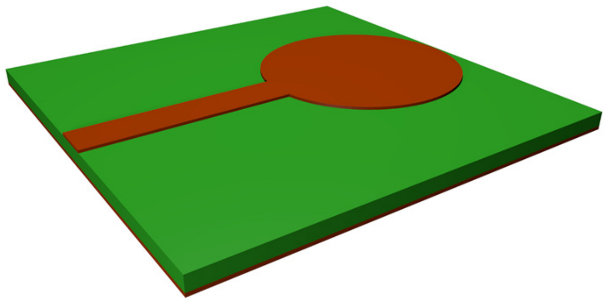
Example (1 of 3)

A 50 Ω microstrip line on FR-4 (ε_r = 4.4) operates at 2.4 GHz and is connected to patch antenna which has a 120 Ω input impedance. How much power is reflected? How can the circuit be improved?



Reflected Power: $\left|\Gamma_L\right|^2 = \left|\frac{Z_L - Z_0}{Z_L - Z_0}\right|^2 = \left|\frac{(120\ \Omega)-(50\ \Omega)}{(120\ \Omega)+(50\ \Omega)}\right|^2 = \left|0.4118\right|^2 = 17\%$

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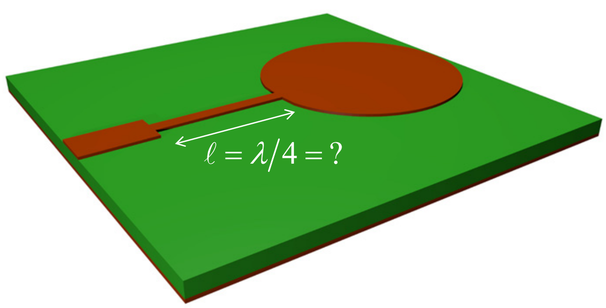
Example (2 of 3)

A 50 Ω microstrip line on FR-4 (ε_r = 4.4) operates at 2.4 GHz and is connected to patch antenna which has a 120 Ω input impedance. How much power is reflected? How can the circuit be improved?



Design: $Z_{ar} = \sqrt{Z_L Z_0} = \sqrt{(120\ \Omega)(50\ \Omega)} = 77.5\ \Omega$

Perform an EM analysis to determine TL dimensions to get 50 Ω. For TEM mode,

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c_0}\sqrt{\mu_r \varepsilon_r} = \frac{2\pi\left(2.4\times10^9\ \text{s}^{-1}\right)}{\left(3.0\times10^8\ \text{m/s}\right)}\sqrt{(1.0)(4.4)} = 105.44\ \text{rad/s}$$

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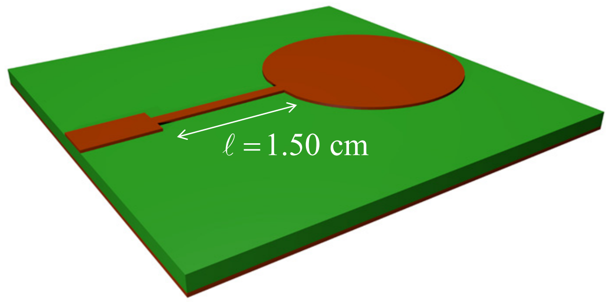
Example (3 of 3)

A 50 Ω microstrip line on FR-4 (ε_r = 4.4) operates at 2.4 GHz and is connected to patch antenna which has a 120 Ω input impedance. How much power is reflected? How can the circuit be improved?



Design: Given β, the length of the line should be

$$\beta_{ar}\ell = \frac{\pi}{2} \quad \rightarrow \quad \ell = \frac{\pi}{2\beta_{ar}} = \frac{\pi}{2(105.44\ \text{rad/s})} = 1.4898\times10^{-2}\ \text{m}$$

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Stubs

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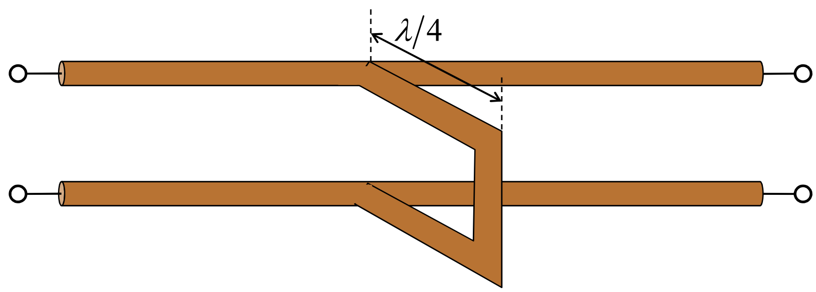
What is a Stub? (1 of 3)



What do short circuits look like $\lambda/4$ away?

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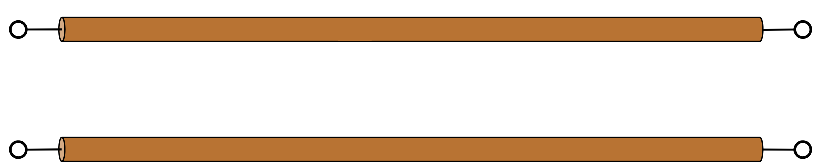
What is a Stub? (2 of 2)



What do short circuits look like $\lambda/4$ away? Open circuits!

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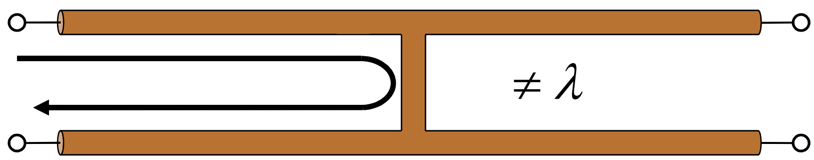
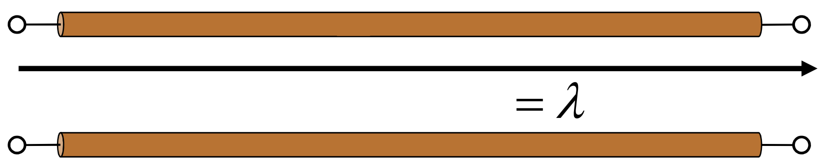
The Shorted Stub is a Band Pass Filter

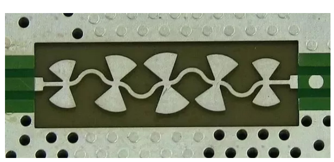
The circuit is actually shorted for all frequencies other than whatever frequency has wavelength λ inside the line. The short circuit blocks all signals.

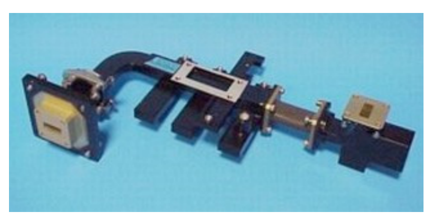


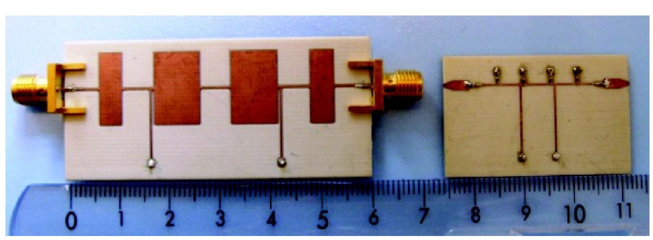
At the frequency with wavelength λ, the circuit is not shorted and signals are allowed to pass.



Stubs in Practice







Scattering Parameters

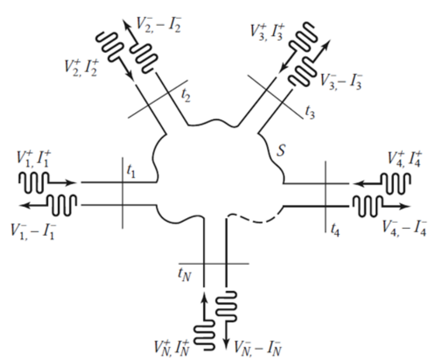
Definition of a Scattering Matrix

The scattering matrix relates the amplitudes of the input waves to the amplitudes of the output waves.

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \\ S_{N1} & S_{N2} & & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

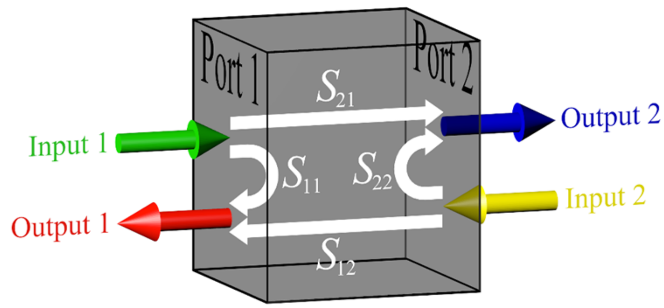
$$S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{\text{no other applied voltages}}$$

N-Port Network



Any linear system can be reduced to a single scattering matrix that describes how it behaves.

S-Matrix for Two-Port Networks



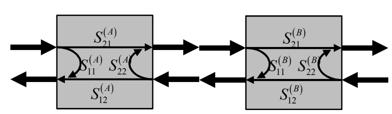
$$\begin{bmatrix} \text{Output 1} \\ \text{Output 2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \text{Input 1} \\ \text{Input 2} \end{bmatrix}$$

S_{11} is synonymous with reflection coefficient.
S_{21} is synonymous with transmission coefficient.

Very often, engineers will say "S-1-1" instead of saying "reflection," and say "S-2-1" instead of saying transmission.

Combining S-Matrices

Suppose there are two circuits, A and B, described by scattering matrices that placed in series. What is the scattering matrix of the combined network?



The answer is NOT matrix multiplication!!!

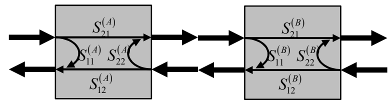
$$\begin{bmatrix} S_{11}^{(AB)} & S_{12}^{(AB)} \\ S_{21}^{(AB)} & S_{22}^{(AB)} \end{bmatrix} \neq \begin{bmatrix} S_{11}^{(A)} & S_{12}^{(A)} \\ S_{21}^{(A)} & S_{22}^{(A)} \end{bmatrix} \begin{bmatrix} S_{11}^{(B)} & S_{12}^{(B)} \\ S_{21}^{(B)} & S_{22}^{(B)} \end{bmatrix}$$

Instead, it is a Redheffer star product.

$$\begin{bmatrix} S_{11}^{(AB)} & S_{12}^{(AB)} \\ S_{21}^{(AB)} & S_{22}^{(AB)} \end{bmatrix} = \begin{bmatrix} S_{11}^{(A)} & S_{12}^{(A)} \\ S_{21}^{(A)} & S_{22}^{(A)} \end{bmatrix} \otimes \begin{bmatrix} S_{11}^{(B)} & S_{12}^{(B)} \\ S_{21}^{(B)} & S_{22}^{(B)} \end{bmatrix}$$

Redheffer Star Product

APPLIED EM



$$\begin{bmatrix} S_{11}^{(AB)} & S_{12}^{(AB)} \\ S_{21}^{(AB)} & S_{22}^{(AB)} \end{bmatrix} = \begin{bmatrix} S_{11}^{(A)} & S_{12}^{(A)} \\ S_{21}^{(A)} & S_{22}^{(A)} \end{bmatrix} \otimes \begin{bmatrix} S_{11}^{(B)} & S_{12}^{(B)} \\ S_{21}^{(B)} & S_{22}^{(B)} \end{bmatrix}$$

$$S_{11}^{(AB)} = \frac{S_{11}^{(A)} - S_{11}^{(A)} S_{22}^{(A)} S_{11}^{(B)} + S_{12}^{(A)} S_{21}^{(A)} S_{11}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$

$$S_{22}^{(AB)} = \frac{S_{22}^{(B)} - S_{22}^{(A)} S_{11}^{(B)} S_{22}^{(B)} + S_{22}^{(A)} S_{12}^{(B)} S_{21}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$

$$S_{12}^{(AB)} = \frac{S_{12}^{(A)} S_{12}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$

$$S_{21}^{(AB)} = \frac{S_{21}^{(A)} S_{21}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$