EE 4347
Applied Electromagnetics

Topic 4g

Analysis & Design of Multi Segment TLs

Lecture Outline

• Circuit/Wave Equivalence
• Analysis of Multi Segment Transmission Lines
• Digital Filter Analogy
• Example
Here we have a “stepped-impedance” microwave circuit.
We view the circuit as a series of discrete segments that are uniform within the segment.

This is equivalent to waves propagating through multiple slabs of dielectric.
Analysis of Multi-Segment Transmission Lines

How do we calculate the reflection from this circuit?

Impedance Transformation Method (1 of 6)
Impedance Transformation Method (2 of 6)

Step 1 – The input impedance $Z_{in}$ at the load is simply the load impedance $Z_L$.

Impedance Transformation Method (3 of 6)

Step 2 – We use impedance transformation to calculate the impedance looking into the fourth segment.

$$Z' = Z_L + jZ_L \tan (\beta d_L)$$

$$Z' = Z_L + jZ_L \tan (\beta_4 d_4)$$
Step 3 – We use impedance transformation to calculate the impedance looking into the third segment.

\[ Z'_3 = Z_3 \frac{Z'_4 + jZ_3 \tan(\beta d_3)}{Z_3 + jZ'_4 \tan(\beta d_3)} \]

Step 4 – We use impedance transformation to calculate the impedance looking into the second segment.

\[ Z'_2 = Z_2 \frac{Z'_3 + jZ_2 \tan(\beta d_2)}{Z_2 + jZ'_3 \tan(\beta d_2)} \]
Step 5 – We use impedance transformation to calculate the impedance looking into the first segment.

\[ Z'_1 = Z_{1g} + jZ_1 \tan (\beta d_1) \]

Step 6 – We calculate the overall reflection as seen by the generator.

\[ \Gamma = \frac{Z'_1 - Z_g}{Z'_1 + Z_g} \]

It is straightforward to extend this procedure to analyze circuits composed of any number of segments.
Impedance Transformation Method for Waves

Impedance Transformation for Waves
\[ \eta' = \eta_i \eta_{i+1} + j \eta_i \tan(\beta d_i) \]
\[ \frac{\eta_{i+1}}{\eta_i} + j \eta_{i+1} \tan(\beta d_i) \]

Overall Reflection
\[ r = \frac{\eta'_{i+1} - \eta_s}{\eta'_{i+1} + \eta_s} \]
\[ R = |r| \]

Notes on the Impedance Transformation Method

• Very fast and simple to implement!
• Difficult to modify the method to calculate transmission when the materials have loss or gain.
• When gain and loss can be ignored, \( R + T = 1 \).
• Method cannot directly visualize the fields inside of the device.
Digital Filter Analogy

Conditions for Analogy

The reflection coefficient $\Gamma_i$ for a wave in medium $i$ incident on medium $i+1$ is given by

$$\Gamma_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i} = \frac{\eta_{i+1} - \eta_i}{\eta_{i+1} + \eta_i}$$

Notice this is written for both transmission lines and dielectric slabs.

The overall reflection coefficient $\Gamma_{in}$ from a series of $M$ segments is very complicated. However, it can be greatly simplified given two conditions:

1. The electrical length of each segment is the same.
   $$\psi \approx \beta_i d_i \approx \cdots \approx \beta_M d_M$$

2. The reflection coefficients $\Gamma_i$ at the interfaces are small, allowing us to ignore waves reflected more than once.
   $$|\Gamma_i| < \sim 0.1$$
Expression for Overall Reflection

Given the two conditions on the previous slide, the overall reflection coefficient $\Gamma_{in}$ from $M$ layers can be written as

$$\Gamma_{in} = \Gamma_0 + \Gamma_1 e^{-j2\psi} + \Gamma_2 e^{-j4\psi} + \cdots + \Gamma_M e^{-j2M\psi}$$

$$\Gamma_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i} = \frac{\eta_{i+1} - \eta_i}{\eta_{i+1} + \eta_i} \quad \psi = \beta_i d_i \equiv \cdots \equiv \beta_M d_M$$

Digital FIR Filters

The general form of the transfer function for a finite impulse response (FIR) digital filter is

$$H(z) = \sum_{k=0}^{M} h(k) z^{-k} = h(0) + h(1) z^{-1} + h(2) z^{-2} + \cdots + h(M) z^{-M}$$

If we excite the digital filter with an impulse function at $t = 0$, the we observe $h(0)$ at time 0, $h(1)$ at time 1, $h(2)$ at time 2, and so on.

The frequency-domain response of a digital FIR filter is

$$H(\omega) = \sum_{k=0}^{M} h(k) e^{-j2k\omega} = h(0) + h(1) e^{-j2\omega} + h(2) e^{-j4\omega} + \cdots + h(M) e^{-j2M\omega}$$
The Analogy

If we compare the overall reflection coefficient $\Gamma_{in}$ of our multi-segment/multi-layer device, we see that it has the same form as a digital filter.

$$\Gamma_{in} = \Gamma_0 + \Gamma_1 e^{-j2\psi} + \Gamma_2 e^{-j4\psi} + \cdots + \Gamma_M e^{-j2M\psi}$$

$$H = h(0) + h(1)e^{-j2\psi} + h(2)e^{-j4\psi} + \cdots + h(M)e^{-j2M\psi}$$

This means we can design multi-segment/multi-layer filters just like we design a digital filter.

Much is known about designing digital filters and all of this knowledge and all of the tools can be used to design filters for electromagnetic waves!

Example
**Step 1 – Analyze TL Regions**

An electromagnetic simulation is performed to calculate the parameters characterizing the transmission lines in each segment of the filter.

- \( L_1 = L_3 = 242 \text{ nH/m} \)
- \( C_1 = C_3 = 96 \text{ pF/m} \)
- \( \beta_1 = \beta_3 = 72.7 \text{ m}^{-1} \)
- \( Z_1 = Z_3 = 50 \Omega \)
- \( L_2 = 153 \text{ nH/m} \)
- \( \beta_2 = 74.4 \text{ m}^{-1} \)
- \( Z_2 = 30 \Omega \)

\( \beta \) reported at 2.4 GHz.
Step 2 – Simulate Multi-Segment Device

Simulation (1D): ~10 sec  Simulation (3D): ~4 min

Each frequency $\omega$ is an entirely different simulation. $\beta$ must be recalculated at each frequency $\omega$.

$$\beta_i = \omega \sqrt{L_i C_i}$$