

EE 4347

## Applied Electromagnetics

Topic 5b

# Parallel Plate Waveguide

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## Lecture Outline

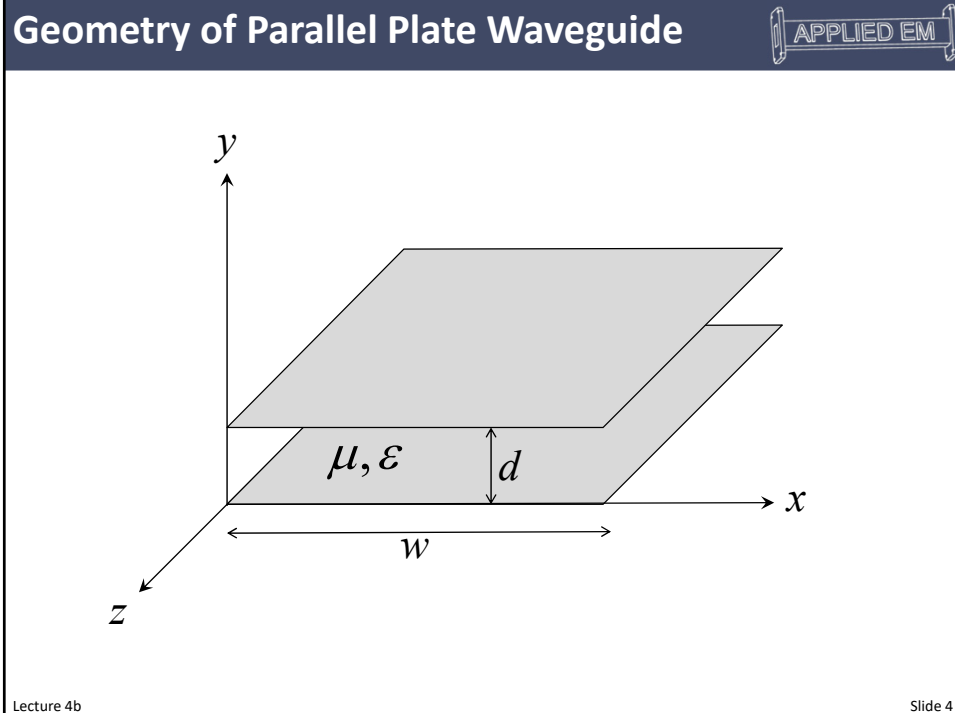


- What is a parallel plate waveguide?
- TEM Analysis
- TM Analysis
- TE Analysis
- Conclusions

# What is a Parallel Plate Waveguide?

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## Notes on the Parallel Plate Waveguide



- Becoming very popular for transmitting differential signals around a circuit board.
- Simply analysis and demonstrates most of the concepts of waveguides.
- Differential lines have confined fields for reduced interference with other devices in close proximity.
- Differential lines exhibit common mode rejection for noise immunity.

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# TEM Analysis

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## Starting Point for TEM Analysis



Assuming the parallel plate waveguide has an LHI dielectric between the plates, we start with the homogeneous Laplace's equation.

$$\nabla^2 V(x, y, z) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

The parallel plate waveguide is uniform in the  $x$  and  $z$  directions so our governing equation reduces to

$$\cancel{\frac{\partial^2 V}{\partial x^2}} + \frac{\partial^2 V}{\partial y^2} + \cancel{\frac{\partial^2 V}{\partial z^2}} = 0$$

$$\frac{\partial^2 V}{\partial y^2} = 0$$

Note, by assuming the field is uniform in the  $x$  direction, we are ignoring the fringing fields at the edges.

$$\frac{d^2 V}{dy^2} = 0$$

The derivative becomes ordinary because  $y$  is the only independent variable left.

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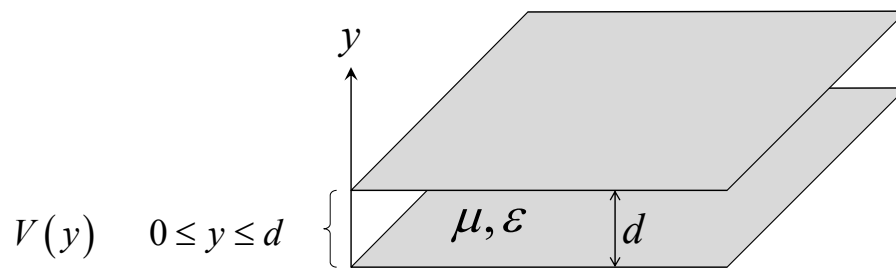
## How to Interpret Governing Equation



Our governing equation is

$$\frac{d^2 V}{dy^2} = 0$$

The solution to this will give us  $V(y)$ .



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## Boundary Conditions



We need boundary conditions to solve our differential equation.

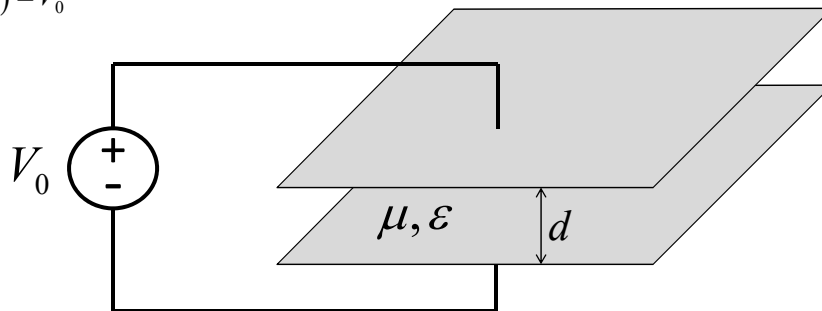
$$V(0) = ?$$

$$V(d) = ?$$

Apply a voltage  $V_0$  across the plates and the boundary conditions will be

$$V(0) = 0$$

$$V(d) = V_0$$



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## General Solution to Differential Equation



Our differential equation with boundary conditions is

$$\frac{d^2V}{dy^2} = 0 \quad 0 \leq y \leq d \quad V(0) = 0 \quad \text{and} \quad V(d) = V_0$$

This is solved by integrating by  $y$  twice.

$$\frac{d^2V}{dy^2} = 0$$

$$\frac{dV}{dy} = A$$

$$V = Ay + B$$

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## Apply Boundary Conditions



Our general solution is

$$V(y) = Ay + B$$

We apply the boundary condition at  $y = 0$ .

$$V(0) = 0$$

$$A \cdot 0 + B = 0$$

$$B = 0$$

We apply the boundary condition at  $y = d$ .

$$V(d) = V_0$$

$$A \cdot d + \cancel{B} = V_0$$

$$A \cdot d = V_0$$

$$A = V_0/d$$

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## The Solution (1 of 2)



The final solution to the governing equation is

$$V(y) = \frac{V_0}{d} y$$

We are still not done because we still do not know much about the waveguide.

The electric field is calculated from the electric potential as

$$\vec{E} = -\nabla V = -\hat{a}_x \frac{\partial V}{\partial x} - \hat{a}_y \frac{\partial V}{\partial y} - \hat{a}_z \frac{\partial V}{\partial z}$$

$$\vec{E} = -\hat{a}_x \frac{\partial}{\partial x} \left( \frac{V_0}{d} y \right) - \hat{a}_y \frac{\partial}{\partial y} \left( \frac{V_0}{d} y \right) - \hat{a}_z \frac{\partial}{\partial z} \left( \frac{V_0}{d} y \right)$$

$$\vec{E} = -\hat{a}_x \frac{\partial}{\partial x} \left( \frac{V_0}{d} y \right) - \hat{a}_y \frac{V_0}{d} - \hat{a}_z \frac{\partial}{\partial z} \left( \frac{V_0}{d} y \right)$$

$$\vec{E} = -\hat{a}_y \frac{V_0}{d}$$

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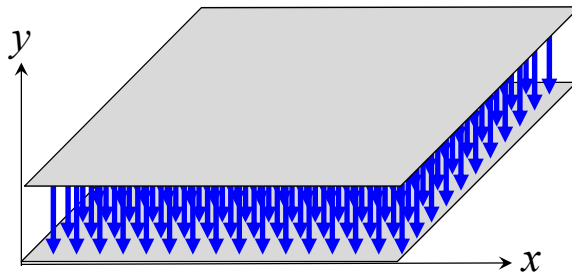
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## The Solution (2 of 2)



If we ignore the fringing fields outside of the waveguide, the electric field is expressed as

$$\vec{E}(x, y) = \begin{cases} -\hat{a}_y \frac{V_0}{d} & \text{for } 0 \leq x \leq w \text{ and } 0 \leq y \leq d \\ 0 & \text{otherwise} \end{cases}$$



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## The Wave Solution



We derived the form of the TEM wave by way of an electrostatic analysis.

This ignores the wave nature of a TEM wave.

To account for the wave nature, we must incorporate a term that accumulates phase in the  $z$  direction.

$$\vec{E}(x, y, z) = \begin{cases} -\hat{a}_y \frac{V_0}{d} e^{-j\beta z} & \text{for } 0 \leq x \leq w \text{ and } 0 \leq y \leq d \\ 0 & \text{otherwise} \end{cases}$$

It follows that the magnetic field component is

$$\vec{H}(x, y, z) = \frac{\hat{a}_z \times \vec{E}}{Z_{\text{TEM}}} = \frac{\hat{a}_z \times \left( -\hat{a}_y \frac{V_0}{d} e^{-j\beta z} \right)}{\eta} = -(\hat{a}_z \times \hat{a}_y) \frac{V_0}{\eta d} e^{-j\beta z} = \hat{a}_x \frac{V_0}{\eta d} e^{-j\beta z}$$

$$\vec{H}(x, y, z) = \begin{cases} \hat{a}_x \frac{V_0}{\eta d} e^{-j\beta z} & \text{for } 0 \leq x \leq w \text{ and } 0 \leq y \leq d \\ 0 & \text{otherwise} \end{cases}$$

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## Impedance from Wave Solution (1 of 2)



The impedance of the TEM wave is defined as

$$Z_{\text{TEM}} = \frac{V_0}{I}$$

We must determine the current term  $I$ . Recall the magnetic field above an infinite current sheet is

$$\vec{H}_{1 \text{ sheet}} = \frac{\vec{K} \times \hat{n}}{2}$$

$\vec{K} \equiv$  surface current density (A/m)       $\hat{n} = -\hat{a}_y$   
Using this equation for the parallel plate waveguide ignores fringing fields at the edges.

It follows that the field between two current sheets (i.e. in our parallel plate waveguide) is

$$\vec{H}_{2 \text{ sheets}} = \vec{K} \times \hat{n}$$

Solving this for the surface current term yields

$$\vec{K} = \hat{n} \times \vec{H} = (-\hat{a}_y) \times \vec{H} = \vec{H} \times \hat{a}_y$$

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## Impedance from Wave Solution (2 of 2)



We can find the total current  $I$  by integrating the surface current across the plate.

$$I = \int_0^w (\vec{K} \cdot \hat{a}_z) dx = \int_0^w [(\vec{H} \times \hat{a}_y) \cdot \hat{a}_z] dx = \int_0^w H_x dx$$

Let  $z = 0$  and our wave solution reduces to

$$H_x(z=0) = \frac{V_0}{\eta d}$$

Substituting this into our equation for  $I$  yields

$$I = \int_0^w \left( \frac{V_0}{\eta d} \right) dx = \frac{V_0}{\eta d} \int_0^w dx = \frac{V_0}{\eta d} w = \frac{w V_0}{d \eta}$$

The characteristic impedance is found by substituting this into the original definition.

$$Z_{\text{TEM}} = \frac{V_0}{I} = \frac{V_0}{\frac{w V_0}{d \eta}} = \eta \frac{d}{w}$$

$$Z_{\text{TEM}} = \eta \frac{d}{w}$$

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## Propagation Constant $\beta$



We cannot calculate a propagation constant  $\beta$  from our solution because we analyzed it using an electrostatic approximation.

TEM waves propagate with nearly the same speed as a plane wave in an infinite medium composed of the dielectric that is between the plates.

$$\beta_{\text{TEM}} \cong \omega\sqrt{\mu\epsilon}$$

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## TE Analysis

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## Recall the Starting Point



The governing equation for TE analysis is

$$\frac{\partial^2 H_{0,z}}{\partial x^2} + \frac{\partial^2 H_{0,z}}{\partial y^2} - k_c^2 H_{0,z} = 0 \quad k_c^2 = k^2 - \beta^2$$

After a solution is obtained, the remaining field components are calculated according to

$$\begin{aligned} H_{0,x} &= -\frac{j\beta}{k_c^2} \frac{\partial H_{0,z}}{\partial x} & E_{0,x} &= -\frac{j\omega\mu}{k_c^2} \frac{\partial H_{0,z}}{\partial y} \\ H_{0,y} &= -\frac{j\beta}{k_c^2} \frac{\partial H_{0,z}}{\partial y} & E_{0,y} &= \frac{j\omega\mu}{k_c^2} \frac{\partial H_{0,z}}{\partial x} \\ & & E_{0,z} &= 0 \end{aligned}$$

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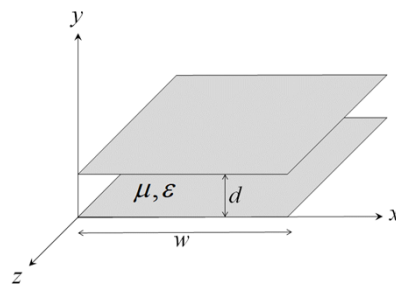
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## Simplify Governing Equation



Assuming the waveguide is uniform in the direction of  $x$

$$\frac{\partial}{\partial x} = 0 \quad \text{and} \quad \frac{\partial^2}{\partial x^2} = 0$$



The governing equation reduces to

$$\cancel{\frac{\partial^2 H_{0,z}}{\partial x^2}} + \frac{\partial^2 H_{0,z}}{\partial y^2} - k_c^2 H_{0,z} = 0 \quad \rightarrow \quad \frac{\partial^2 H_{0,z}}{\partial y^2} - k_c^2 H_{0,z} = 0$$

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## General Solution



The general solution to the governing equation is

$$\frac{\partial^2 H_{0,z}}{\partial y^2} - k_c^2 H_{0,z} = 0 \quad \rightarrow \quad H_{0,z} = A \sin(k_c y) + B \cos(k_c y)$$

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## Boundary Conditions (1 of 2)



The electric field must be zero at the plates.

The solution, however, is in terms of the magnetic field.

We must write it in terms of an electric field.

The only component of the electric field tangential to the interface is  $E_{0,x}$ .

$$\begin{aligned} E_{0,x} &= -\frac{j\omega\mu}{k_c^2} \frac{\partial H_{0,z}}{\partial y} = -\frac{j\omega\mu}{k_c^2} \frac{\partial}{\partial y} [A \sin(k_c y) + B \cos(k_c y)] \\ &= -\frac{j\omega\mu}{k_c} [A \cos(k_c y) - B \sin(k_c y)] \end{aligned}$$

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## Boundary Conditions (2 of 2)



The first boundary condition is

$$E_{0,x}(x,0) = 0$$

$$E_{0,x}(x,0) = -\frac{j\omega\mu}{k_c} [A \cos(0) - B \sin(0)] = -\frac{j\omega\mu}{k_c} A = 0 \quad \rightarrow \quad A = 0$$

The second boundary condition is

$$E_{0,x}(x,d) = 0$$

$$E_{0,x}(x,d) = -\frac{j\omega\mu}{k_c} [-B \sin(k_c d)] = B \frac{j\omega\mu}{k_c} \sin(k_c d)$$

We cannot choose  $B = 0$  because that would lead to a trivial solution. Instead, it must be the sine term that is zero at  $y = d$ .

$$\sin(k_c d) = 0 \quad \rightarrow \quad k_c d = n\pi \quad n = 1, 2, 3, \dots$$

The cutoff wave number is then

$$k_c = \frac{n\pi}{d} \quad n = 1, 2, 3, \dots$$

Note that  $n = 0$  would force the entire field to be zero so this is not a valid solution.

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## Phase Constant



Recall our definition of the cutoff wave number. Solve this for  $\beta$ .

$$k_c^2 = k^2 - \beta^2 \quad \rightarrow \quad \beta = \sqrt{k^2 - k_c^2}$$

We now have an expression for  $k_c$  that arose from the boundary conditions.

$$\beta = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2} \quad n = 1, 2, 3, \dots$$

We see that we have an infinite number of discrete solutions where the order of the mode is  $n$ .

$$\beta_n = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2} \quad n = 1, 2, 3, \dots$$

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## Final Solution



Recall the general solution was

$$\frac{\partial^2 H_{0,z}}{\partial y^2} - k_c^2 H_{0,z} = 0 \quad \rightarrow \quad H_{0,z} = A \sin(k_c y) + B \cos(k_c y)$$

But now we know that  $A = 0$  and  $k_c = n\pi/d$ . The final solution is

$$H_{0,z}(x, y) = B_n \cos\left(\frac{n\pi y}{d}\right) \quad \rightarrow \quad H_z(x, y, z) = B_n \cos\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z}$$

The remaining field components are calculated from this result.

$$\begin{aligned} H_x &= -\frac{j\beta}{k_c^2} \frac{\partial H_z}{\partial x} = -\frac{j\beta}{k_c^2} \frac{\partial}{\partial x} \left[ B_n \cos\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z} \right] = 0 \\ H_y &= -\frac{j\beta}{k_c^2} \frac{\partial H_z}{\partial y} = -\frac{j\beta}{k_c^2} \frac{\partial}{\partial y} \left[ B_n \cos\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z} \right] = \frac{j\beta}{k_c} B_n \sin\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z} \\ E_x &= -\frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y} = -\frac{j\omega\mu}{k_c^2} \frac{\partial}{\partial y} \left[ B_n \cos\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z} \right] = \frac{j\omega\mu}{k_c} B_n \sin\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z} \\ E_y &= \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x} = \frac{j\omega\mu}{k_c^2} \frac{\partial}{\partial x} \left[ B_n \cos\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z} \right] = 0 \\ E_z &= 0 \end{aligned}$$

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## Cutoff Condition



Recall that we calculate our phase constant as

$$\beta_n = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2} \quad n = 1, 2, 3, \dots$$

This becomes imaginary when  $k_c > k$ . Values of  $n$  that cause this condition correspond to modes that are "cutoff." These are still modes, but they decay very quickly so they are almost never used.

$$k_c = \omega_c \sqrt{\mu\epsilon} = \frac{n\pi}{d}$$

$$2\pi f_c \sqrt{\mu\epsilon} = \frac{n\pi}{d}$$

$$f_c = \frac{n}{2d\sqrt{\mu\epsilon}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} \quad \longrightarrow \quad \text{This is the cutoff frequency for the TM}_n \text{ mode.}$$

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## Characteristic Impedance



The characteristic impedance is defined as

$$Z_{\text{TE}} = \frac{E_{0,x}}{H_{0,y}} = -\frac{E_{0,y}}{H_{0,x}}$$

We substitute in our expressions for the field quantities to obtain

$$Z_{\text{TE}} = \frac{E_{0,x}}{H_{0,y}} = \frac{\frac{j\omega\mu}{k_c} B_n \sin\left(\frac{n\pi y}{d}\right)}{\frac{j\beta}{k_c} B_n \sin\left(\frac{n\pi y}{d}\right)} = \frac{\omega\mu}{\beta} = \eta \frac{k}{\beta}$$

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## TM Analysis

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## Recall the Starting Point



The governing equation for TM analysis is

$$\frac{\partial^2 E_{0,z}}{\partial x^2} + \frac{\partial^2 E_{0,z}}{\partial y^2} - k_c^2 E_{0,z} = 0 \quad k_c^2 = k^2 - \beta^2$$

After a solution is obtained, the remaining field components are calculated according to

$$\begin{aligned} H_{0,x} &= \frac{j\omega\epsilon}{k_c^2} \frac{\partial E_{0,z}}{\partial y} & E_{0,x} &= -\frac{j\beta}{k_c^2} \frac{\partial E_{0,z}}{\partial x} \\ H_{0,y} &= -\frac{j\omega\epsilon}{k_c^2} \frac{\partial E_{0,z}}{\partial x} & E_{0,y} &= -\frac{j\beta}{k_c^2} \frac{\partial E_{0,z}}{\partial y} \\ H_{0,z} &= 0 \end{aligned}$$

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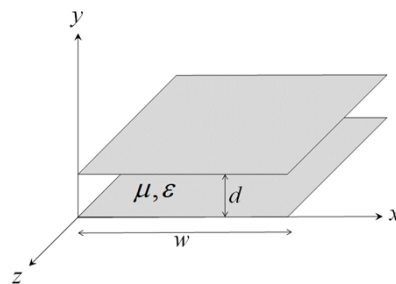
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## Simplify Governing Equation



Assuming the waveguide is uniform in the direction of  $x$

$$\frac{\partial}{\partial x} = 0 \quad \text{and} \quad \frac{\partial^2}{\partial x^2} = 0$$



The governing equation reduces to

$$\cancel{\frac{\partial^2 E_{0,z}}{\partial x^2}} + \frac{\partial^2 E_{0,z}}{\partial y^2} - k_c^2 E_{0,z} = 0 \quad \rightarrow \quad \frac{\partial^2 E_{0,z}}{\partial y^2} - k_c^2 E_{0,z} = 0$$

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## General Solution



The general solution to the governing equation is

$$\frac{\partial^2 E_{0,z}}{\partial y^2} - k_c^2 E_{0,z} = 0 \quad \rightarrow \quad E_{0,z} = A \sin(k_c y) + B \cos(k_c y)$$

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## Boundary Conditions



The electric field component  $E_{0,z}$  is tangential to the interfaces to the boundary conditions are applied to this directly.

The first boundary condition is

$$E_{0,z}(x, 0) = A \sin(0) + B \cos(0) = B = 0 \quad \rightarrow \quad B = 0$$

The second boundary condition is

$$E_{0,z}(x, d) = A \sin(k_c d) = 0$$

We cannot choose  $A = 0$  because that would lead to a trivial solution. Instead, it must be sine term that is zero at  $y = d$ .

$$\sin(k_c d) = 0 \quad \rightarrow \quad k_c d = n\pi \quad n = 0, 1, 2, 3, \dots$$

The cutoff wave number is then

$$k_c = \frac{n\pi}{d} \quad n = 0, 1, 2, 3, \dots$$

Note that  $n = 0$  is allowed in this case because it does not force the field to be zero. It does, however, force the field to be uniform. Thus  $TM_0$  is the TEM mode.

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## Phase Constant



Recall our definition of the cutoff wave number. Solve this for  $\beta$ .

$$k_c^2 = k^2 - \beta^2 \quad \rightarrow \quad \beta = \sqrt{k^2 - k_c^2}$$

We now have an expression for  $k_c$  that arose from the boundary conditions.

$$\beta = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2} \quad n = 0, 1, 2, 3, \dots$$

We see that we have an infinite number of discrete solutions where the order of the mode is  $n$ .

$$\beta_n = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2} \quad n = 1, 2, 3, \dots$$

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## Final Solution



Recall the general solution was

$$\frac{\partial^2 E_{0,z}}{\partial y^2} - k_c^2 E_{0,z} = 0 \quad \rightarrow \quad E_{0,z} = A \sin(k_c y) + B \cos(k_c y)$$

But now we know that  $B = 0$  and  $k_c = n\pi/d$ . The final solution is

$$E_{0,z}(x, y) = A_n \sin\left(\frac{n\pi y}{d}\right) \quad \rightarrow \quad E_z(x, y, z) = A_n \sin\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z}$$

The remaining field components are calculated from this result.

$$H_x = \frac{j\omega\epsilon}{k_c^2} \frac{\partial E_z}{\partial y} = \frac{j\omega\epsilon}{k_c^2} \frac{\partial}{\partial y} \left[ A_n \sin\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z} \right] = \frac{j\omega\epsilon}{k_c} A_n \cos\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z}$$

$$H_y = -\frac{j\omega\epsilon}{k_c^2} \frac{\partial E_z}{\partial x} = -\frac{j\omega\epsilon}{k_c^2} \frac{\partial}{\partial x} \left[ A_n \sin\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z} \right] = 0$$

$$H_z = 0$$

$$E_x = -\frac{j\beta}{k_c^2} \frac{\partial E_z}{\partial x} = -\frac{j\beta}{k_c^2} \frac{\partial}{\partial x} \left[ A_n \sin\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z} \right] = 0$$

$$E_y = -\frac{j\beta}{k_c^2} \frac{\partial E_z}{\partial y} = -\frac{j\beta}{k_c^2} \frac{\partial}{\partial y} \left[ A_n \sin\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z} \right] = -\frac{j\beta}{k_c} A_n \cos\left(\frac{n\pi y}{d}\right) e^{-j\beta_n z}$$

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## Cutoff Condition



Recall that we calculate our phase constant as

$$\beta_n = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2} \quad n = 0, 1, 2, 3, \dots$$

This becomes imaginary when  $k_c > k$ . Values of  $n$  that cause this condition correspond to modes that are “cutoff.” These are still modes, but they decay very quickly so they are almost never used.

$$k_c = \omega_c \sqrt{\mu\epsilon} = \frac{n\pi}{d}$$

$$2\pi f_c \sqrt{\mu\epsilon} = \frac{n\pi}{d}$$

$$f_c = \frac{n}{2d\sqrt{\mu\epsilon}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} \quad \longrightarrow \quad \text{This is the cutoff frequency for the } \text{TM}_n \text{ mode.}$$

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## Characteristic Impedance



The characteristic impedance is defined as

$$Z_{\text{TM}} = \frac{E_{0,x}}{H_{0,y}} = -\frac{E_{0,y}}{H_{0,x}}$$

We substitute in our expressions for these field quantities to obtain

$$Z_{\text{TM}} = -\frac{E_{0,y}}{H_{0,x}} = -\frac{-\frac{j\beta}{k_c} A_n \cos\left(\frac{n\pi y}{d}\right)}{\frac{j\omega\epsilon}{k_c} A_n \cos\left(\frac{n\pi y}{d}\right)} = \frac{\beta}{\omega\epsilon} = \eta \frac{\beta}{k}$$

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# Conclusion

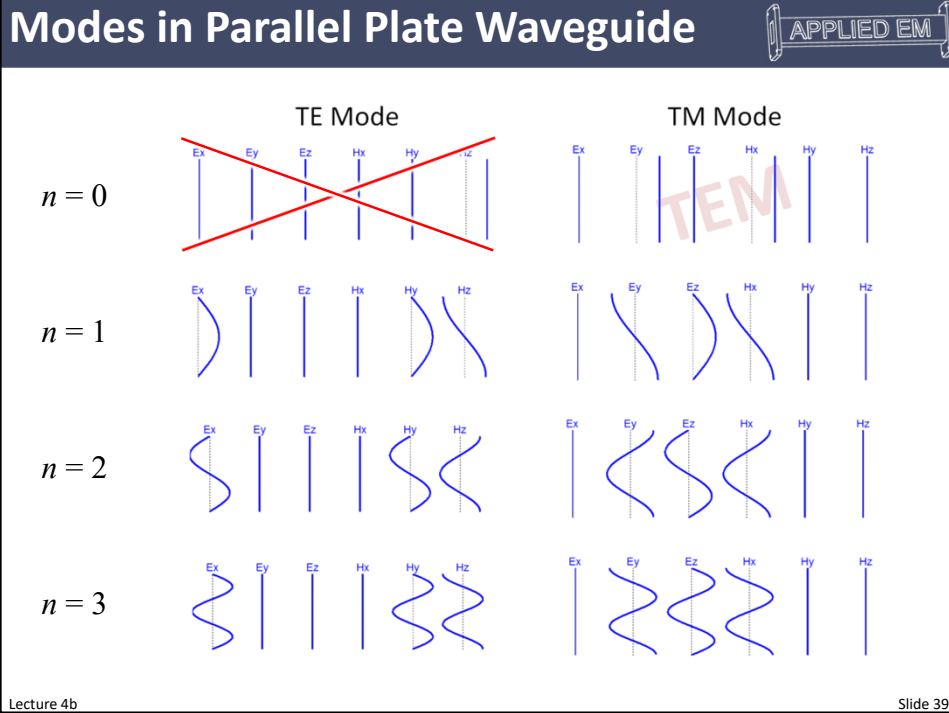
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Summary of Parallel Plate Waveguide <span style="float: right;">APPLIED EM</span>			
Quantity	TEM Mode	$n = 0, 1, 2, 3, \dots$ TM <sub>n</sub> Mode	$n = 1, 2, 3, \dots$ TE <sub>n</sub> Mode
$k$	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$
$k_c$	0	$n\pi/d$	$n\pi/d$
$\beta$	$k = \omega\sqrt{\mu\epsilon}$	$\sqrt{k^2 - k_c^2}$	$\sqrt{k^2 - k_c^2}$
$\lambda_c$	$\infty$	$2\pi/k_c = 2d/n$	$2\pi/k_c = 2d/n$
$\lambda_g$	$2\pi/k$	$2\pi/\beta$	$2\pi/\beta$
$v_p$	$\omega/k = 1/\sqrt{\mu\epsilon}$	$\omega/\beta$	$\omega/\beta$
$\alpha_d$	$(k \tan \delta)/2$	$(k^2 \tan \delta)/2\beta$	$(k^2 \tan \delta)/2\beta$
$\alpha_c$	$R_s/\eta d$	$2kR_s/\beta\eta d$	$2k_c^2 R_s/k\beta\eta d$
$E_z$	0	$A \sin(n\pi y/d)e^{-j\beta z}$	0
$H_z$	0	0	$B \cos(n\pi y/d)e^{-j\beta z}$
$E_x$	0	0	$(j\omega\mu/k_c)B \sin(n\pi y/d)e^{-j\beta z}$
$E_y$	$(-V_o/d)e^{-j\beta z}$	$(-j\beta/k_c)A \cos(n\pi y/d)e^{-j\beta z}$	0
$H_x$	$(V_o/\eta d)e^{-j\beta z}$	$(j\omega\epsilon/k_c)A \cos(n\pi y/d)e^{-j\beta z}$	0
$H_y$	0	0	$(j\beta/k_c)B_n \sin(n\pi y/d)e^{-j\beta z}$
$Z$	$Z_{\text{TEM}} = \eta d/W$	$Z_{\text{TM}} = \beta\eta/k$	$Z_{\text{TE}} = k\eta/\beta$

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## Notes

APPLIED EM

- Supports TEM modes because it has two conductors.
- Supports TE and TM modes when it has a homogeneous dielectric
- The lowest order mode is  $TM_0$  which is the TEM mode.

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