MICROWAVE ENGINEERING

Problem #1: Complex Propagation Constant

The complex propagation constant γ is related to the lumped element transmission line parameters R, L, G, and C through the following equation.

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

From this equation, derive the following expressions for the attenuation coefficient α and the propagation constant β in terms of just R, L, G, and C.

$$\alpha = \sqrt{\frac{1}{2}\left[RG - \omega^2 LC + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}\right]}$$

$$\beta = \sqrt{\frac{1}{2}\left[-RG + \omega^2 LC + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}\right]}$$

Problem #2: Complex Impedance

The transmission line equations can be written as

$$-\frac{dV(z)}{dz} = (R + j\omega L)I(z) \qquad -\frac{dI(z)}{dz} = (G + j\omega C)V(z)$$

Given that the definition of the characteristic impedance of a transmission line is $Z_0 = V_0^+ / I_0^+$, derive the following expression for Z_0 in terms of just R, L, G, and C.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Problem #3: Impedance Transformation

A lossless transmission line with C = 24 pF/m and L = 60 nH/m operates at 5.0 GHz and is connected to a 100 nF capacitor as the load.

a. What is the input impedance of the transmission line if its length is 4.0 cm?
b. Is the input impedance inductive, capacitive, or resistive?
c. What value inductor, capacitor, or resistor does the input impedance act like?

Problem #4: Reflections

A 50 Ω transmission line is connected to an antenna with a 72 Ω input impedance.

a. What is the reflection coefficient from the antenna?
b. What fraction of power is fed into the antenna?
c. What is the voltage standing wave ratio (VSWR) in the transmission line feeding the antenna?
d. What can be done to deliver power to the antenna more efficiently? List as many solutions as possible and be specific.