Problem #1: Equations for Waveguide Analysis

Part (a)

The general framework for waveguide analysis places the cross section of the waveguide in the x/y plane (i.e. transverse plane), while the longitudinal direction is along the z-axis. Starting with Maxwell's curl equations

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$
$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E},$$

derive expressions for the transverse field components (i.e. E_x, E_y, H_x, and H_y) in terms of just the longitudinal field components (i.e. E_z and H_z). Show all work. Assume the waveguide modes have the general form

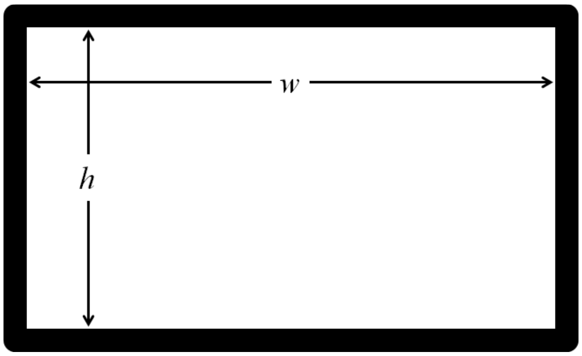
$$\vec{E}(x,y,z) = \vec{E}_0(x,y)e^{-j\beta z} \qquad \vec{H}(x,y,z) = \vec{H}_0(x,y)e^{-j\beta z}$$

Part (b)

Derive expressions for the wave equations for the longitudinal field components E_z and H_z. Clearly identify any approximations you make.

Problem #2: Rectangular Waveguide

A WR 430 waveguide is made from a highly conductive metal and is specified to operate from 1.72 GHz up to 2.60 GHz. A cross section of the waveguide and its inner dimensions are shown below. For the WR 430, the dimensions are w = 4.3 inches and h = 2.15 inches.



a. What are the cutoff frequencies for the first and second order modes? Which modes are these?
b. What is the propagation constant β at 2.0 GHz?
c. What is the length of a $\lambda/4$ section of this waveguide at this same frequency?
d. How many electromagnetic modes exist at 3.0 GHz?
e. Considering the cutoff frequencies calculated above, why is the waveguide specified over the range 1.72 to 2.60 GHz?