MICROWAVE ENGINEERING

ECE 4380/5390

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Topic 4

Numerical Analysis of Transmission Lines

Outline

- Solution Approach
- Using `tlcalc.p` to Analyze Transmission Lines

Solution Approach

Maxwell's Equations

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We start with Maxwell's equations.

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \bullet \vec{D} = 0$$

$$\nabla \times \vec{H} = \vec{J} + \partial \vec{D} \big/ \partial t$$

$$\nabla \times \vec{E} = -\partial \vec{B} \big/ \partial t$$

Small Size Approximation

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The dimensions of a transmission are typically much smaller than the operating wavelength so the wave nature of electromagnetics is less important to consider. Therefore, we are essentially solving Maxwell's equations as $d/dt \rightarrow 0$.

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \bullet \vec{D} = 0$$

$$\nabla \times \vec{H} = \vec{J} + \cancel{\partial \vec{D} / \partial t}$$

$$\nabla \times \vec{E} = \cancel{-\partial \vec{B} / \partial t}$$

Electrostatics & Magnetostatics

Maxwell's equations have decoupled into two sets of equations. Once describes electrostatics while the other describes magnetostatics.

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \bullet \vec{D} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \times \vec{E} = 0$$

Electrostatics

$$\nabla \bullet \vec{D} = 0$$

$$\nabla \times \vec{E} = 0$$

Magnetostatics

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

Governing Equations

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Maxwell's equations:

$$\nabla \bullet \vec{D} = 0 \qquad \nabla \times \vec{E} = 0$$

In addition, we have the constitutive relation

$$\vec{D} = \varepsilon_r \vec{E}$$

We do not like to solve vector equations if we do not have to. Electrostatic fields are completely characterized by the scalar potential V.

$$\vec{E} = -\nabla V$$

Differential Equation to Solve

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1. Substitute Eq. (3) into Eq. (1) to eliminate D.

$$\nabla \bullet \left(\varepsilon_r \vec{E} \right) = 0 \qquad \text{Eq. (5)}$$

2. Substitute Eq. (4) into Eq. (5) to eliminated E.

$$\boxed{\nabla \bullet \left[\varepsilon_r \left(\nabla V \right) \right] = 0} \qquad \text{Eq. (6)}$$

$$\nabla \bullet \vec{D} = 0 \qquad \text{Eq. (1)}$$

$$\nabla \times \vec{E} = 0 \qquad \text{Eq. (2)}$$

$$\vec{D} = \varepsilon_r \vec{E} \qquad \text{Eq. (3)}$$

$$\vec{E} = -\nabla V \qquad \text{Eq. (4)}$$

Derive Finite-Difference Equation

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When the dielectric is homogeneous, our differential equation simplifies to

$$\nabla^2 V = 0$$

In Cartesian coordinates, this expands to

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

This equation is approximated with finite-differences

$$\frac{V(i+1,j) - 2V(i,j) + V(i-1,j)}{(\Delta x)^2} + \frac{V(i,j+1) - 2V(i,j) + V(i,j-1)}{(\Delta y)^2} = 0$$

Write Large Set of Equations

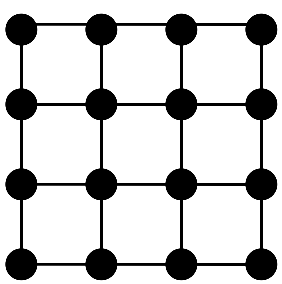
We rearrange our finite-difference equation

$$\left[\frac{1}{(\Delta x)^2}\right]V(i-1,j)+\left[\frac{1}{(\Delta x)^2}\right]V(i+1,j)-\left[\frac{2}{(\Delta x)^2}+\frac{2}{(\Delta y)^2}\right]V(i,j)+\left[\frac{1}{(\Delta y)^2}\right]V(i,j-1)+\left[\frac{1}{(\Delta y)^2}\right]V(i,j+1)=0$$

This equation is written once for every point on the grid.

$$N_x = 4 \qquad \Delta x = 0.5$$

$$N_y = 4 \qquad \Delta y = 0.5$$

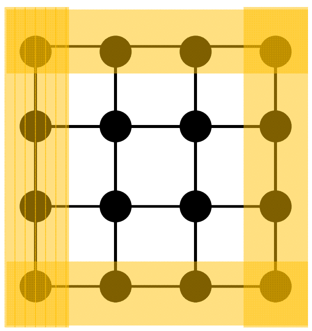


$$\begin{aligned}
4\cdot V(0,1)+4\cdot V(2,1)+16\cdot V(1,1)+4\cdot V(1,0)+4\cdot V(1,2)&=0\\
4\cdot V(1,1)+4\cdot V(3,1)+16\cdot V(2,1)+4\cdot V(2,0)+4\cdot V(2,2)&=0\\
4\cdot V(2,1)+4\cdot V(4,1)+16\cdot V(3,1)+4\cdot V(3,0)+4\cdot V(3,2)&=0\\
4\cdot V(3,1)+4\cdot V(5,1)+16\cdot V(4,1)+4\cdot V(4,0)+4\cdot V(4,2)&=0\\
4\cdot V(0,2)+4\cdot V(2,2)+16\cdot V(1,2)+4\cdot V(1,1)+4\cdot V(1,3)&=0\\
4\cdot V(1,2)+4\cdot V(3,2)+16\cdot V(2,2)+4\cdot V(2,1)+4\cdot V(2,3)&=0\\
4\cdot V(2,2)+4\cdot V(4,2)+16\cdot V(3,2)+4\cdot V(3,1)+4\cdot V(3,3)&=0\\
4\cdot V(3,2)+4\cdot V(5,2)+16\cdot V(4,2)+4\cdot V(4,1)+4\cdot V(4,3)&=0\\
4\cdot V(0,3)+4\cdot V(2,3)+16\cdot V(1,3)+4\cdot V(1,2)+4\cdot V(1,4)&=0\\
4\cdot V(1,3)+4\cdot V(3,3)+16\cdot V(2,3)+4\cdot V(2,2)+4\cdot V(2,4)&=0\\
4\cdot V(2,3)+4\cdot V(4,3)+16\cdot V(3,3)+4\cdot V(3,2)+4\cdot V(3,4)&=0\\
4\cdot V(3,3)+4\cdot V(5,3)+16\cdot V(4,3)+4\cdot V(4,2)+4\cdot V(4,4)&=0\\
4\cdot V(0,4)+4\cdot V(2,4)+16\cdot V(1,4)+4\cdot V(1,3)+4\cdot V(1,5)&=0\\
4\cdot V(1,4)+4\cdot V(3,4)+16\cdot V(2,4)+4\cdot V(2,3)+4\cdot V(2,5)&=0\\
4\cdot V(2,4)+4\cdot V(4,4)+16\cdot V(3,4)+4\cdot V(3,3)+4\cdot V(3,5)&=0\\
4\cdot V(3,4)+4\cdot V(5,4)+16\cdot V(4,4)+4\cdot V(4,3)+4\cdot V(4,5)&=0
\end{aligned}$$

Consider Boundary Conditions

$N_x = 4 \qquad \Delta x = 0.5$

$N_y = 4 \qquad \Delta y = 0.5$



$$
\begin{aligned}
4 \cdot V(0,1) + 4 \cdot V(2,1) + 16 \cdot V(1,1) + 4 \cdot V(1,0) + 4 \cdot V(1,2) &= 0 \\
4 \cdot V(1,1) + 4 \cdot V(3,1) + 16 \cdot V(2,1) + 4 \cdot V(2,0) + 4 \cdot V(2,2) &= 0 \\
4 \cdot V(2,1) + 4 \cdot V(4,1) + 16 \cdot V(3,1) + 4 \cdot V(3,0) + 4 \cdot V(3,2) &= 0 \\
4 \cdot V(3,1) + 4 \cdot V(5,1) + 16 \cdot V(4,1) + 4 \cdot V(4,0) + 4 \cdot V(4,2) &= 0 \\
4 \cdot V(0,2) + 4 \cdot V(2,2) + 16 \cdot V(1,2) + 4 \cdot V(1,1) + 4 \cdot V(1,3) &= 0 \\
4 \cdot V(1,2) + 4 \cdot V(3,2) + 16 \cdot V(2,2) + 4 \cdot V(2,1) + 4 \cdot V(2,3) &= 0 \\
4 \cdot V(2,2) + 4 \cdot V(4,2) + 16 \cdot V(3,2) + 4 \cdot V(3,1) + 4 \cdot V(3,3) &= 0 \\
4 \cdot V(3,2) + 4 \cdot V(5,2) + 16 \cdot V(4,2) + 4 \cdot V(4,1) + 4 \cdot V(4,3) &= 0 \\
4 \cdot V(0,3) + 4 \cdot V(2,3) + 16 \cdot V(1,3) + 4 \cdot V(1,2) + 4 \cdot V(1,4) &= 0 \\
4 \cdot V(1,3) + 4 \cdot V(3,3) + 16 \cdot V(2,3) + 4 \cdot V(2,2) + 4 \cdot V(2,4) &= 0 \\
4 \cdot V(2,3) + 4 \cdot V(4,3) + 16 \cdot V(3,3) + 4 \cdot V(3,2) + 4 \cdot V(3,4) &= 0 \\
4 \cdot V(3,3) + 4 \cdot V(5,3) + 16 \cdot V(4,3) + 4 \cdot V(4,2) + 4 \cdot V(4,4) &= 0 \\
4 \cdot V(0,4) + 4 \cdot V(2,4) + 16 \cdot V(1,4) + 4 \cdot V(1,3) + 4 \cdot V(1,5) &= 0 \\
4 \cdot V(1,4) + 4 \cdot V(3,4) + 16 \cdot V(2,4) + 4 \cdot V(2,3) + 4 \cdot V(2,5) &= 0 \\
4 \cdot V(2,4) + 4 \cdot V(4,4) + 16 \cdot V(3,4) + 4 \cdot V(3,3) + 4 \cdot V(3,5) &= 0 \\
4 \cdot V(3,4) + 4 \cdot V(5,4) + 16 \cdot V(4,4) + 4 \cdot V(4,3) + 4 \cdot V(4,5) &= 0
\end{aligned}
$$

We will set all of the highlighted terms to zero

→ Dirichlet Boundary Conditions

Build Matrix

$$
\begin{aligned}
& && + 4 \cdot V(2,1) + 16 \cdot V(1,1) && && + 4 \cdot V(1,2) = 0 \\
& 4 \cdot V(1,1) && + 4 \cdot V(3,1) + 16 \cdot V(2,1) && && + 4 \cdot V(2,2) = 0 \\
& 4 \cdot V(2,1) && + 4 \cdot V(4,1) + 16 \cdot V(3,1) && && + 4 \cdot V(3,2) = 0 \\
& 4 \cdot V(3,1) && \qquad\qquad\quad + 16 \cdot V(4,1) && && + 4 \cdot V(4,2) = 0 \\
& && + 4 \cdot V(2,2) + 16 \cdot V(1,2) && + 4 \cdot V(1,1) && + 4 \cdot V(1,3) = 0 \\
& 4 \cdot V(1,2) && + 4 \cdot V(3,2) + 16 \cdot V(2,2) && + 4 \cdot V(2,1) && + 4 \cdot V(2,3) = 0 \\
& 4 \cdot V(2,2) && + 4 \\
& 4 \cdot V(3,2) \\
& && + 4 \\
& 4 \cdot V(1,3) && + 4 \\
& 4 \cdot V(2,3) && + 4 \\
& 4 \cdot V(3,3) \\
& && + 4 \\
& 4 \cdot V(1,4) && + 4 \\
& 4 \cdot V(2,4) && + 4 \\
& 4 \cdot V(3,4)
\end{aligned}
$$

$$
\begin{bmatrix}
-16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & -16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & -16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 4 & -16 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & -16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16
\end{bmatrix}
\begin{bmatrix}
V(1,1) \\ V(2,1) \\ V(3,1) \\ V(4,1) \\ V(1,2) \\ V(2,2) \\ V(3,2) \\ V(4,2) \\ V(1,3) \\ V(2,3) \\ V(3,3) \\ V(4,3) \\ V(1,4) \\ V(2,4) \\ V(3,4) \\ V(4,4)
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

Is Our Matrix Equation Solvable?

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$$
\underbrace{\begin{bmatrix}
-16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 4 & -16 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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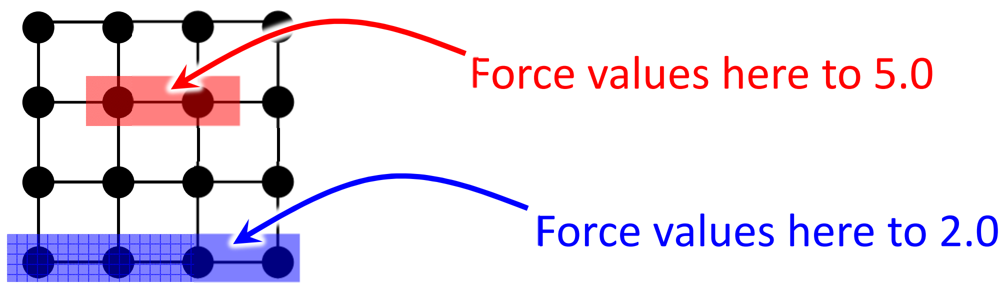
0 & 4 & 0 & 0 & 4 & -16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 4 & 0 & 0 & 4 & -16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 4 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 4 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 0 & 0 & 0 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -16 & 4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16
\end{bmatrix}}_{[L]}
\underbrace{\begin{bmatrix}
V(1,1) \\ V(2,1) \\ V(3,1) \\ V(4,1) \\ V(1,2) \\ V(2,2) \\ V(3,2) \\ V(4,2) \\ V(1,3) \\ V(2,3) \\ V(3,3) \\ V(4,3) \\ V(1,4) \\ V(2,4) \\ V(3,4) \\ V(4,4)
\end{bmatrix}}_{[v]}
=
\underbrace{\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}}_{[0]}
$$

$$\nabla^2 V = 0 \quad \rightarrow \quad [L][v] = [0] \quad \rightarrow \quad [v] = [L]^{-1}[0] = [0]$$

Trivial Solution

Force Known Potentials



$V(2,2)=5$

$V(3,2)=5$

$V(1,4)=2$

$V(2,4)=2$

$V(3,4)=2$

$V(4,4)=2$

$$\begin{bmatrix}
-16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & -16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 4 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 0 & 0 & 0 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} V(1,1) \\ V(2,1) \\ V(3,1) \\ V(4,1) \\ V(1,2) \\ V(2,2) \\ V(3,2) \\ V(4,2) \\ V(1,3) \\ V(2,3) \\ V(3,3) \\ V(4,3) \\ V(1,4) \\ V(2,4) \\ V(3,4) \\ V(4,4) \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

Solve for Scalar Potential

$$
\underbrace{\begin{bmatrix}
-16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & -16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & -16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 4 & -16 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & -16 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 4 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 4 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & -16 & 0 & 0 & 0 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}}_{[L]}
\underbrace{\begin{bmatrix}
V(1,1) \\ V(2,1) \\ V(3,1) \\ V(4,1) \\ V(1,2) \\ V(2,2) \\ V(3,2) \\ V(4,2) \\ V(1,3) \\ V(2,3) \\ V(3,3) \\ V(4,3) \\ V(1,4) \\ V(2,4) \\ V(3,4) \\ V(4,4)
\end{bmatrix}}_{[v]}
=
\underbrace{\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 2 \\ 2 \\ 2
\end{bmatrix}}_{[b]}
$$

$$
[L][v]=[b] \quad \rightarrow \quad [v]=[L]^{-1}[b]
$$

Calculating the Fields

Once the scalar potential is solved, the electric field intensity is

$$\vec{E}(x,y) = -\nabla V(x,y)$$

The electric flux density is

$$\vec{D}(x,y) = \varepsilon_r(x,y)\vec{E}(x,y)$$

Distributed Capacitance

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In the electrostatic approximation, the transmission line is a capacitor. The total energy U stored in a capacitor is

$$U = \tfrac{1}{2} \iint_A \left(\vec{D} \bullet \vec{E} \right) dA$$

The capacitance C is related to the total stored energy U through

$$U = \frac{C V_0^2}{2}$$

V_0 is the voltage across the capacitor.

If we set the above equations equal and solve for C, we get

$$\boxed{C = \frac{1}{V_0^2} \iint_A \left(\vec{D} \bullet \vec{E} \right) dA}$$

Distributed Inductance

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The voltage along the transmission line travels at the same velocity as the electric field so we can write

$$v_V = v_E \quad \rightarrow \quad \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} \quad \rightarrow \quad LC = \frac{\mu_r \varepsilon_r}{c_0^2}$$

Solving this for L we get

$$L = \frac{\mu_r \varepsilon_r}{c_0^2 C}$$

This means we can calculate the distributed inductance L directly from the distributed capacitance.

Dielectric materials should not alter the inductance. However if we use the value of *C* calculated on the previous slide, it will. This is incorrect. The solution is to calculate capacitance C_H with air dielectric and then calculate inductance from this.

$$\boxed{L = \frac{\mu_r}{c_0^2 C_H}}$$

Calculating the Transmission Line Parameters

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The characteristic impedance is calculated from the distributed inductance and capacitance through

$$Z_c = \sqrt{\frac{L}{C}}$$

The phase constant is

$$\beta = \omega\sqrt{LC} = k_0 n_{\text{eff}}$$

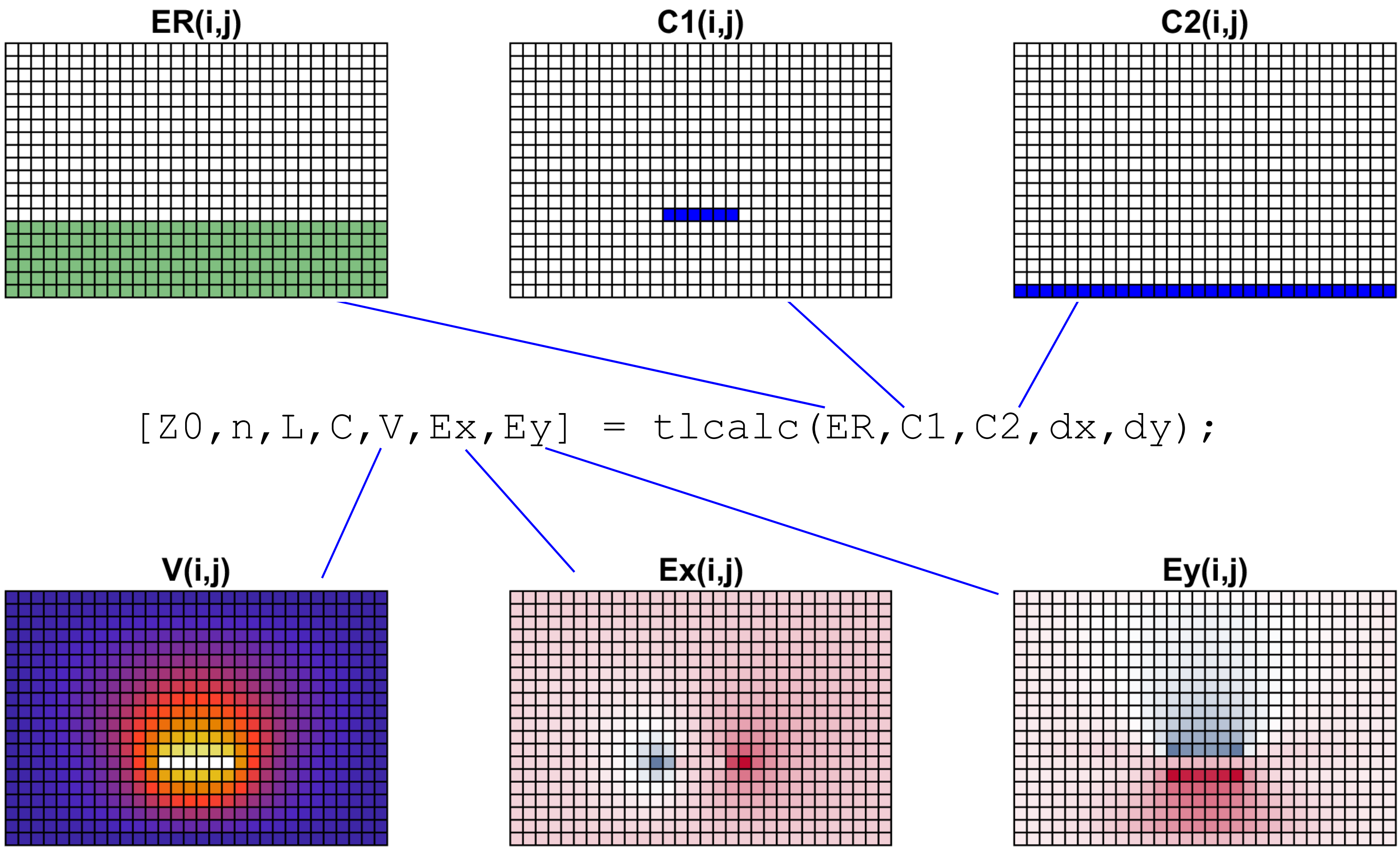
$$k_0 = \frac{2\pi}{\lambda_0}$$

Recall, both Z_c and β are needed to analyze transmission line circuits.

Note: It was assumed $R = G = 0$ (lossless transmission line)

Using `tlcalc.p` to Analyze Transmission Lines

The Short Story



tlcalc() (1 of 2)

```
tlcalc    Transmission Line Calculator

 [Z0,n,L,C,V,Ex,Ey] = tlcalc(ER,C1,C2,dx,dy);

 This MATLAB function calculates the properties of an
 arbitrary transmission.

 ER is a 2D array describing the relative permittivity.
 C1 is a 2D array identifying the points of conductor 1.
 C2 is a 2D array identifying the points of conductor 2.
 dx and dy are the grid resolution parameters.

 To simulate the line C1 is set to 1 volt and C2 is set to 0 volts.

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```
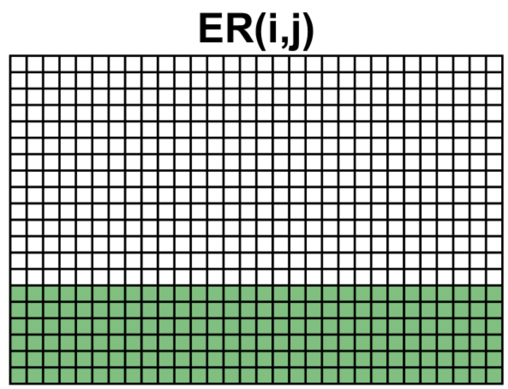
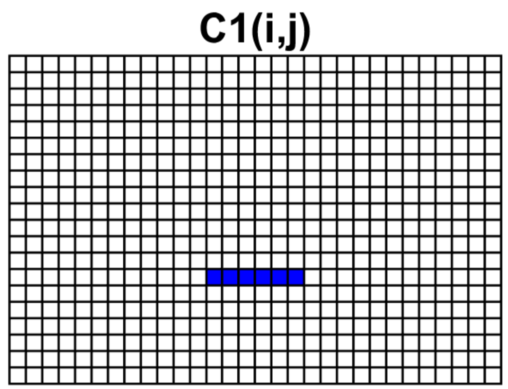
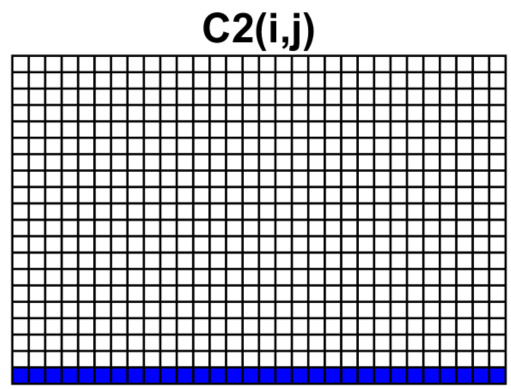

Number Values for Inputs







Permittivity Function

$$1 \le \varepsilon_r(x,y) < \infty$$

ε_r should be purely real.

Loss would be accounted for differently.

Conductor #1

$$0 \equiv \text{dielectric}$$

$$1 \equiv \text{metal}$$

Array should be all 0's except 1's in the positions that describe the first conductor.

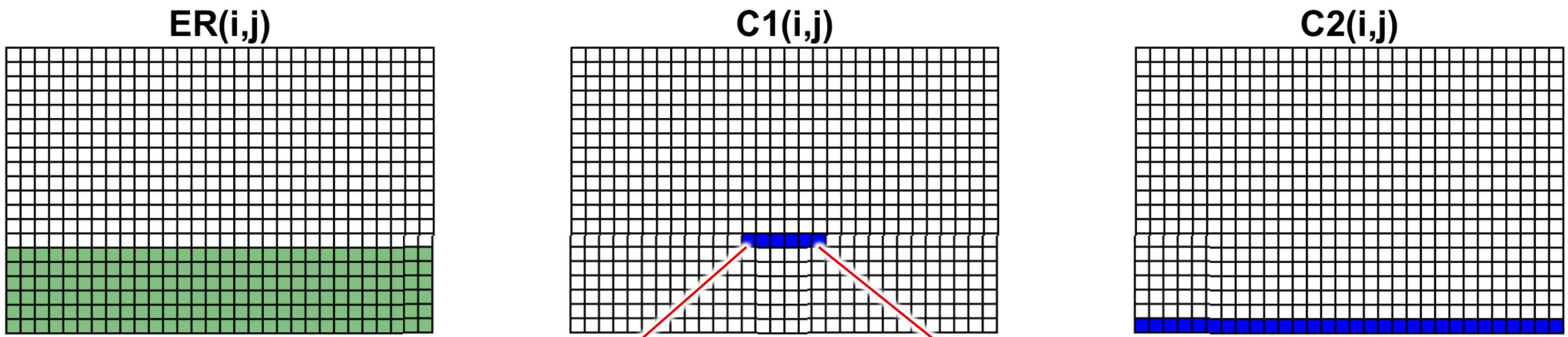
Conductor #2

$$0 \equiv \text{dielectric}$$

$$1 \equiv \text{metal}$$

Array should be all 0's except 1's in the positions that describe the second conductor.

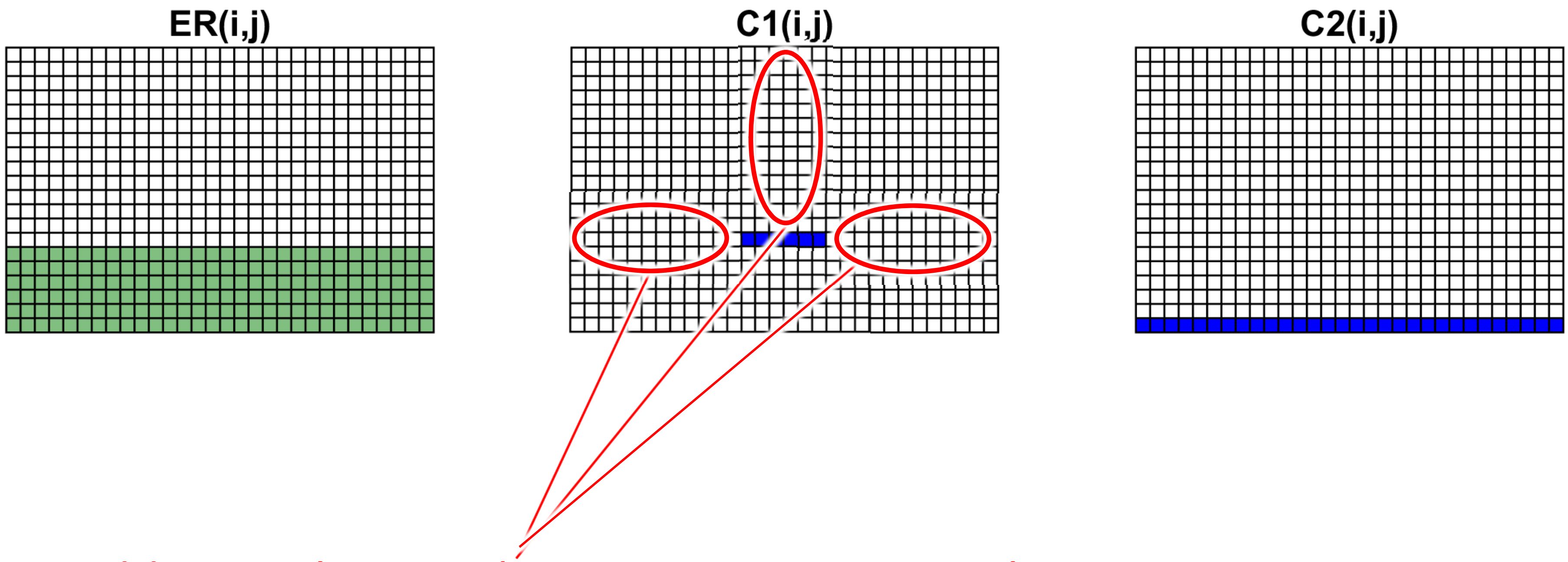
Grid Resolution



Pick your smallest dimension and resolve this with at least 5 - 10 points.

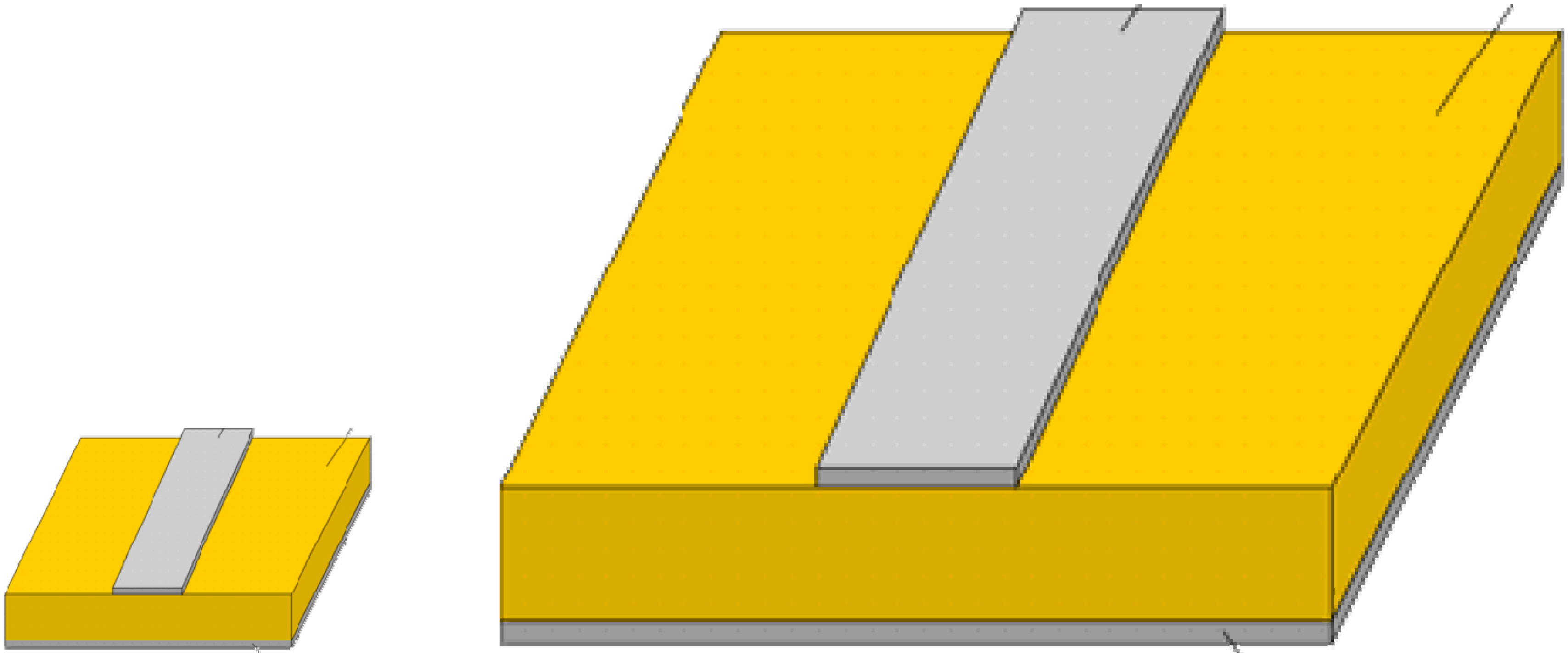
```
dx = w/10;
dy = dx;
```

Space Around Transmission Line



Add enough space between transmission line and the boundary to ensure the electric field decays sufficiently. Remember, we have used Dirichlet boundary conditions.

Scalability



These TLs will have the exact same properties.

Electrostatics as no fundamental size scale.