Antenna Parameters: A Practical Approach

Radiation Pattern and Beamwidth

1. - Plot in polar form the following functions for radiation patterns. You can help yourself with the diagram provided below.
   
   a. \( U(\theta) = \sin^2(\theta) \)
   
   b. \( U(\theta) = \sin(\phi)\cos(\theta) \), \( 0 < \phi < 2\pi \)
   
   c. \( U(\theta) = \cos^2(\theta)\sin(2\theta) \)
   
   d. \( U(\theta) = \sin^2(\theta)\cos(\theta + \frac{\pi}{2}) \)

2. – For the radiation patterns below, find also the half-power beamwidth, both in radians and degrees, and plot it on the top of part 1.
### Polarization

For the next wave functions, determine if the polarization is linear, circular (Right-Hand or Left-Hand), or elliptical. Be careful, recognize that some waves are in time-domain and others in frequency-domain (phasors). Justify your answers.

a. $\vec{E}(x, y, z, t) = (3\hat{x} - j2\hat{y}) \cos(\omega t - kz)$

b. $\vec{E}(x, y, z, t) = (\hat{x} + \hat{y}) \cos(\omega t - kz)$

c. $\vec{E}(z) = (3\hat{x} + 3e^{j\pi/2}\hat{y}) e^{-jkz}$

d. $\vec{E}(z) = (\hat{x} - j\hat{y}) e^{-jkz}$

### Antenna Design

You are an antenna engineer and you are asked to design a high directivity/gain antenna for a space-borne communication system operating at 10 GHz. The specifications of the antenna are such that its pattern consists of one major lobe, and ideally no minor lobes. If they are minor lobes, they are from such low intensity compared to the major lobe that they can be neglected. The pattern is also requested to be symmetrical to the azimuthal plane. In order to meet the requirements, the antenna must have a half-power beamwidth of 10 degrees. To expedite the design, the radiation pattern will be approximated by the function

$$U(\theta, \phi) = \cos^n(\theta)$$

and exists only on the upper hemisphere ($0 \leq \theta \leq \frac{\pi}{2}, \ 0 < \phi < 2\pi$)

a. Determine the value of n (not necessarily an integer, keep 5 significant figures in your calculations)

b. Determine the exact maximum directivity, both unitless and in dB

**HINT:** Approximate the value of n by setting the value of $\cos^n\left(\frac{1}{2} HPBW\right)$ to $\frac{1}{2}$ (effectively computing the Half-Power Beamwidth). Then find the total radiated power $P_{rad}$, which then can be used to find the maximum directivity $D$. 
A communications satellite is in synchronous (stationary) orbit around the earth (assume a stable altitude of 22,370 miles), and it transmits with 8.0 W of power. The receiving antenna is a 210 ft. paraboloidal antenna at the NASA tracking station at Goldstone, CA. Assume the transmitter antenna is isotropic, that both antennas are impedance matched, and resistive losses are negligible. The only mismatch is in the polarization, which was calculated to have an 8-degree tilt between the 2 antennas.

a. Calculate the power density (in $\frac{W}{m^2}$) incident on the receiving antenna.

b. Calculate the power received by the ground-based antenna, if its gain is 60 dB.

**HINT:** Use the Friis transmission formula. The propagation loss factor is the incident power density.