Topic 2 – Antenna Parameters and Figures of Merit (FOM) Continued

EE-4382/5306 - Antenna Engineering

Outline

- Polarization
- Input Impedance
- Radiation Efficiency
- Friis Transmission Equation

Constantine A. Balanis, *Antenna Theory: Analysis and Design* 4th Ed., Wiley, 2016.
Stutzman, Thiele, *Antenna Theory and Design* 3rd Ed., Wiley, 2012.

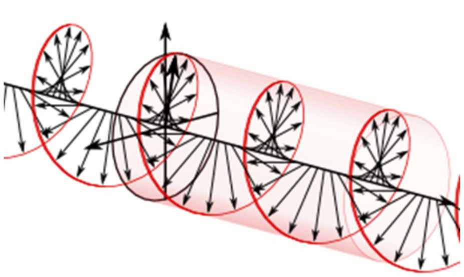
Polarization

Antenna Polarization

It describes the movement of the locus (end point) of the electric field. Polarization can be linear, circular, or elliptical.

Almost every wave is elliptically polarized. There are no such waves that exhibit strictly linear or circular polarization, but simplifications can be made.

Polarization is viewed when it travels away from the observer.



Antenna Polarization

A wave traveling in the -z direction can be written as

$$\widetilde{\mathrm{E}}(z,t) = \hat{\boldsymbol{a}}_{\boldsymbol{x}} E_x(z,t) + \hat{\boldsymbol{a}}_{\boldsymbol{y}} E_y(z,t)$$

$$E_x(z,t) = \mathrm{Re}\left[E_{x0} e^{j(\omega t + kz + \phi_x)}\right] = E_{x0}\cos(\omega t + kz + \phi_x)$$

$$E_y(z,t) = \mathrm{Re}\left[E_{y0} e^{j(\omega t + kz + \phi_y)}\right] = E_{y0}\cos(\omega t + kz + \phi_y)$$

The phase difference between the x and y components of the wave is

$$\Delta\phi = \phi_y - \phi_x$$

$$E_y(z,t) = E_{y0}\cos(\omega t + kz + \Delta\phi)$$

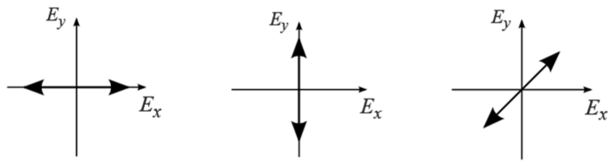
The polarization can be found by calculating the magnitudes of the different components as a function of time and tracing the locus over space.

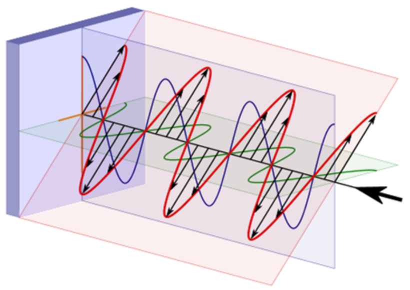
Antenna Polarization

Linear Polarization

The phase difference between components is 0 or multiples of π.

$$\Delta\phi = n\pi, \qquad n = 0,1,2,3,\ldots$$





Antenna Polarization

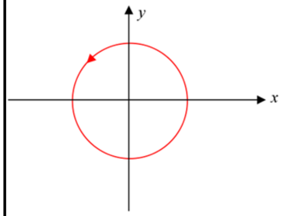
Circular Polarization

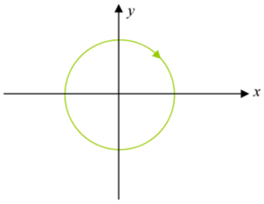
The magnitude of the components is the same.

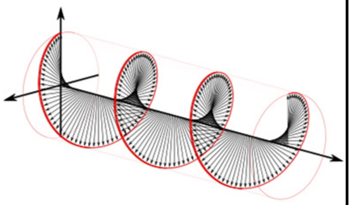
The phase difference between components is odd multiples of $\frac{\pi}{2}$.

Two circular polarizations: Right-Hand Circular and Left-Hand Circular.

$$\Delta\phi = \begin{cases} +\left(\frac{1}{2}+2n\right)\pi\ (\mathrm{RHC}) \\ -\left(\frac{1}{2}+2n\right)\pi\ (\mathrm{LHC}) \end{cases}, \qquad n = 0,1,2,3,\ldots$$







Antenna Polarization

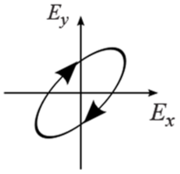
Elliptical Polarization

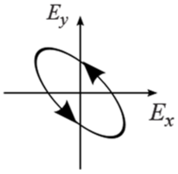
“Not Linear, not circular”

The magnitude of the components is different and the phase difference between components is odd multiples of $\frac{\pi}{2}$.

The phase difference between components is not an odd multiple of $\frac{\pi}{2}$.

Two elliptical polarizations: Right-Hand Circular and Left-Hand Circular.





Antenna Polarization - Example

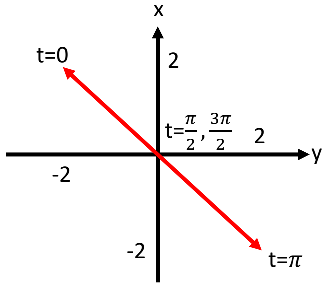
For the following wave, determine the polarization.

$$\vec{E}(z,t) = 2\cos(\omega t - kz)\,\hat{\mathbf{x}} + 2\cos(\omega t - kz + \pi)\,\hat{\mathbf{y}}$$

Solution: Get the vector components of the wave, and trace the wave as a function of time

$$\vec{E} = \mathrm{E_x} + \mathrm{E_y}$$
$$E_x = 2\cos(\omega t - kz)\,\hat{\mathbf{x}}$$
$$E_y = -2\cos(\omega t - kz)\,\hat{\mathbf{y}}$$

t	E_x	E_y
0	2	-2
$\pi/2$	0	0
π	-2	2
$\frac{3\pi}{2}$	0	0



Polarization is linear

Antenna Polarization - Example

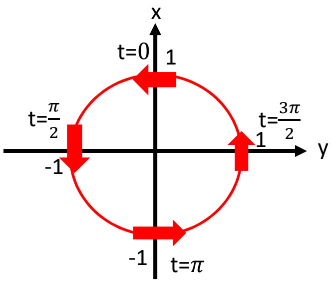
For the following wave, determine the polarization.

$$\tilde{\mathrm{E}}(z) = (\hat{\mathbf{x}} + \mathrm{j}\hat{\mathbf{y}})e^{jkz}$$

Solution: Convert phasor to time-domain, and trace the wave as a function of time

$$\tilde{E} = \mathrm{E_x} + \mathrm{E_y}$$
$$E_x = \mathrm{Re}\left(\left[\hat{\mathbf{x}}e^{jkz}\right]\right) = \cos(\omega t - kz)\,\hat{\mathbf{x}}$$
$$E_y = \mathrm{Re}\left(\left[j\hat{\mathbf{y}}e^{jkz}\right]\right) = \cos\left(\omega t - kz + \frac{\pi}{2}\right)\hat{\mathbf{y}} = -\sin(\omega t - kz)\,\hat{\mathbf{y}}$$

t	E_x	E_y
0	1	0
$\pi/2$	0	-1
π	-1	0
$\frac{3\pi}{2}$	0	1



Polarization is Right-Hand Circular

Polarization Loss Factor and Efficiency

Sometimes the polarization of the transmitting antenna does not match the polarization of the receiving antenna. This is called polarization mismatch. The polarization loss factor quantifies the loss caused by the polarization mismatch.

The incident antenna has the polarization form

$$\mathbf{E}_\mathrm{i} = \hat{\boldsymbol{\rho}}_i E_i$$

The receiving antenna has the polarization form

$$\mathbf{E}_\mathrm{r} = \hat{\boldsymbol{\rho}}_r E_r$$

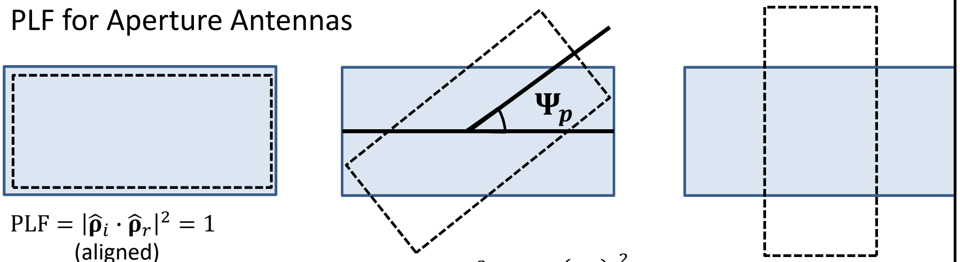
The polarization loss factor is defined as

$$\mathrm{PLF} = |\hat{\boldsymbol{\rho}}_i \cdot \hat{\boldsymbol{\rho}}_r|^2 = |\cos(\Psi_p)|^2$$

Where Ψ_p is the angle between the two vectors.

Polarization Loss Factor and Efficiency

PLF for Aperture Antennas

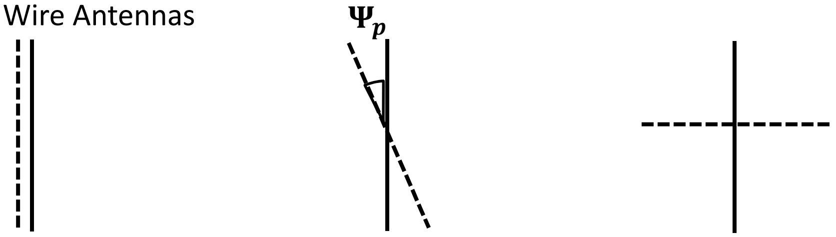


$\mathrm{PLF} = |\hat{\boldsymbol{\rho}}_i \cdot \hat{\boldsymbol{\rho}}_r|^2 = 1$
(aligned)

$\mathrm{PLF} = |\hat{\boldsymbol{\rho}}_i \cdot \hat{\boldsymbol{\rho}}_r|^2 = |\cos(\Psi_p)|^2$
(rotated)

$\mathrm{PLF} = |\hat{\boldsymbol{\rho}}_i \cdot \hat{\boldsymbol{\rho}}_r|^2 = 0$
(orthogonal)

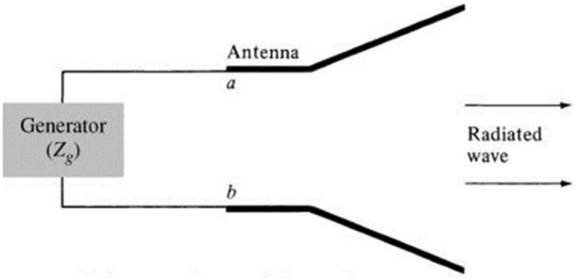
PLF for Wire Antennas



Input Impedance

Input Impedance

It is the impedance presented by the antennas at its terminals.
It is the ratio of voltage to current at a pair of terminals.
It is the ratio of the components of the electric and magnetic fields at a point.



(a) Antenna in transmitting mode

Fig. 2.27a

Chapter 2
Fundamental Parameters

Input Impedance

Thevenin Equivalent: Transmitting

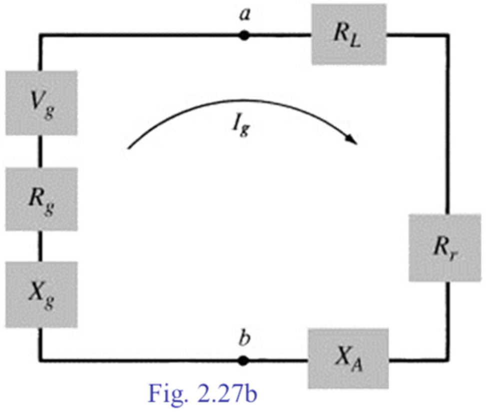


Fig. 2.27b

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Fundamental Parameters

Input Impedance

The impedance of the antenna at the terminals is

$$Z_A = R_A + jX_A$$

Where the resistive (real) part of the antenna consists of two components:

$$R_A = R_r + R_L$$

R_r- Radiation Resistance
R_L- Radiation Loss

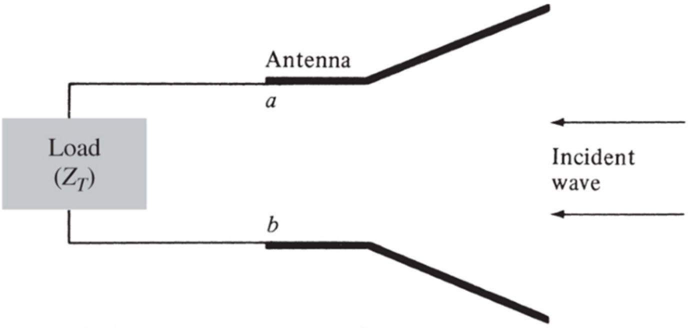
All the related circuit analysis techniques and calculations are used to obtain maximum power transfer (conjugate matching):

$$R_r + R_L = R_g$$
$$X_A = -X_g$$

$$P_s = \frac{|V_g|^2}{4}\left[\frac{1}{R_r+R_L}\right]$$ (Power supplied when conjugate matched)

Input Impedance

Antenna In Receiving Mode



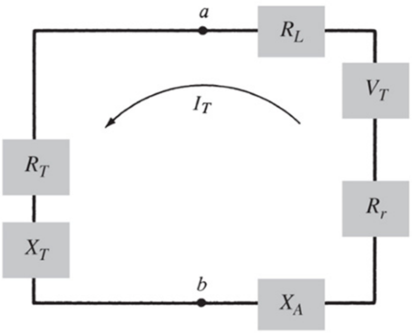
(a) Antenna in receiving mode

Fig. 2.28a

Chapter 2
Fundamental Parameters

Input Impedance

Thevenin Equivalent
(Receiving Mode)



(b) Thevenin equivalent

Fig. 2.28b

Chapter 2
Fundamental Parameters

Radiation Efficiency

Radiation Efficiency

It is also the conduction-dielectric efficiency.

Takes into account the antenna efficiency in terms of antenna impedance (radiation resistance and loss resistance).

This is because conduction-dielectric losses are very difficult to calculate and measure separately.

$$\boxed{e_{cd} = \frac{R_r}{R_r + R_L}}$$

Friis Transmission Equation

Friis Transmission Equation

Relates the power received to the power transmitted between two antennas that are placed by a distance $R > \frac{2D^2}{\lambda}$, where D is the maximum dimension of either antenna.

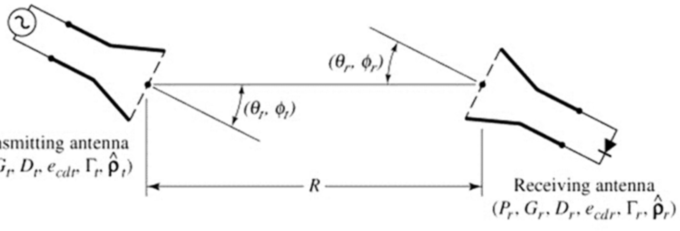


Fig. 2.31

Friis Transmission Equation

$$\frac{P_r}{P_t} = e_{cdt}e_{cdr}\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t,\phi_t)D_r(\theta_r,\phi_r)|\hat{\rho}_t\cdot\hat{\rho}_r|^2$$

e_{cdt}: Antenna efficiency of transmitting antenna

e_{cdr}: Antenna efficiency of receiving antenna

$\left(1-|\Gamma_t|^2\right)$: Reflection efficiency of transmitting antenna

$\left(1-|\Gamma_r|^2\right)$: Reflection efficiency of receiving antenna

$\left(\frac{\lambda}{4\pi R}\right)^2$: Propagation loss factor

$D_t(\theta_t,\phi_t)$: Directivity of transmitting antenna

$D_r(\theta_r,\phi_r)$: Directivity of receiving antenna

$|\hat{\rho}_t\cdot\hat{\rho}_r|^2$: polarization loss factor

Friis Transmission Equation - Example

Example 2.6 (page 89 Balanis): Two lossless X-band ($8.2 - 12.4$ GHz) antennas are separated by a distance of 100λ. The reflection coefficients at the terminals of the transmitting and receiving antennas are 0.1 and 0.2, respectively. The maximum directivity of the transmitting and receiving antennas (over isotropic) are 16 dB and 20 dB, respectively. Assuming that the input power in the lossless transmission line connected to the transmitting antenna is 2 W, and the antennas are aligned for maximum radiation between them and polarization matched, find the power delivered to the load of the receiver.

Solution:

$e_{cdt} = e_{cdr} = 1$ – Antennas are lossless

$|\hat{\rho}_t\cdot\hat{\rho}_r|^2 = 1$ - Polarization Matched

$D_t(\theta_t,\phi_t) = D_{0t}$ - Antennas are aligned for maximum directivity

$D_r(\theta_r,\phi_r) = D_{0r}$ - Antennas are aligned for maximum directivity

$$D_{0t} = 16\text{ dB} = 39.81\text{ (dimensionless)}$$

$$D_{0t} = 20\text{ dB} = 100\text{ (dimensionless)}$$

Friis Transmission Equation - Example

Using Friis Transmission Equation, we obtain

$$P_r = e_{cdt} e_{cdr} \left(1 - |\Gamma_t|^2\right)\left(1 - |\Gamma_r|^2\right)\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) |\hat{\rho}_t \cdot \hat{\rho}_r|^2 P_t$$

$$P_r = (1)(1)\left(1 - (0.1)^2\right)\left(1 - (0.2)^2\right)\left(\frac{\lambda}{4\pi 100\lambda}\right)^2 (39.81)(100)(1)(2)$$

$$\boxed{P_r = 4.777\ \mathrm{mW}}$$