PROPAGATING UNCERTAINTY

EE 4386/5301 Computational Methods for EE

Pioneering 21st Century Electromagnetics and Photonics

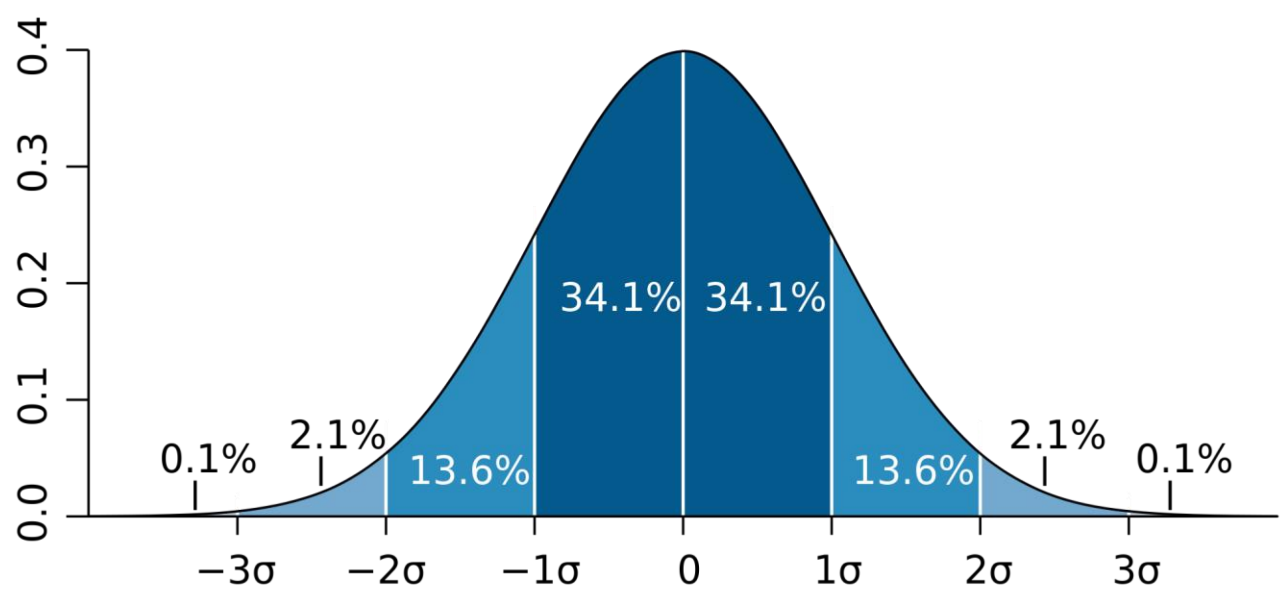
http://emlab.utep.edu/

Description of the Problem

Very often we do not know quantities exactly because there is uncertainty in the measurements. For example, the height of a person might be written as

$$h = 2.8 \pm 0.02 \text{ m}$$

In this case, the uncertainty in the measurement is 0.02 m. We interpret this as the standard deviation σ of a normal distribution.



Propagating Uncertainty

Suppose we wish to calculate some quantity that is a function of multiple variables, each having its own uncertainty.

$$f(x_1, x_2, \cdots, x_M)$$

What is the overall uncertainty for σ_f?

$$\sigma_f^2 = \left(\sigma_1 \frac{\partial f}{\partial x_1}\right)^2 + \left(\sigma_2 \frac{\partial f}{\partial x_2}\right)^2 + \cdots + \left(\sigma_M \frac{\partial f}{\partial x_M}\right)^2$$

Table of Equations

Function	Uncertainty
$f = ax$	$\sigma_f = a\sigma_x$
$f = ax \pm by$	$\sigma_f^2 = (a\sigma_x)^2 + (b\sigma_y)^2$
$f = xy$	$(\sigma_f/f)^2 = (\sigma_x/x)^2 + (\sigma_y/y)^2$
$f = x/y$	$(\sigma_f/f)^2 = (\sigma_x/x)^2 + (\sigma_y/y)^2$
$f = x^{\pm b}$	$\sigma_f/f = b\sigma_x/x$
$f = \ln(\pm bx)$	$\sigma_f = b\sigma_x/x$
$f = \log x$	$\sigma_f = b\sigma_x/(x \ln 10)$
$f = e^{\pm bx}$	$\sigma_f/f = b\sigma_x$
$f = a^{\pm bx}$	$\sigma_f/f = b\sigma_x \ln a$
$f = \sin x$	$\sigma_f = \sigma_x \cos x$
$f = \cos x$	$\sigma_f = \sigma_x \sin x$
$f = \tan x$	$\sigma_f = \sigma_x/\cos^2 x$
$f = \sin^{-1}(x)$	$\sigma_f^2 = \sigma_x^2/(1-x^2)$
$f = \cos^{-1}(x)$	$\sigma_f^2 = \sigma_x^2/(1-x^2)$
$f = \tan^{-1}(x)$	$\sigma_f = \sigma_x/(1+x^2)$