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Computational Methods in EE

Topic 7b – Numerical Analysis of Slab Waveguides

EE 4386/5301 Computational Methods in EE

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Outline

Computational Methods in EE

- Slab Waveguides
- Formulation
- Solution
- Implementation in MATLAB
- More About Resolution and Spacer Regions

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Computational
Methods in EE

Slab Waveguides

3

Refractive Index n

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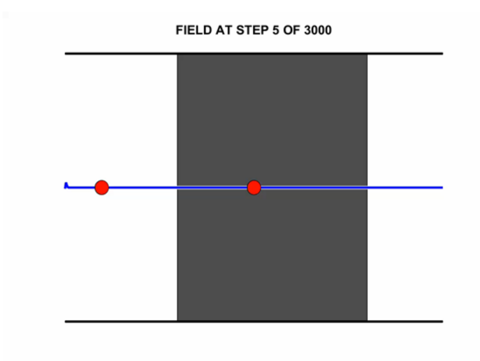
Light travels at different speeds when it is inside different materials.



Frequency is constant.
Speed changes.
Wavelength changes.

The factor by which light slows down is called the *refractive index*.

$$n = \frac{c}{v}$$

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Snell's Law

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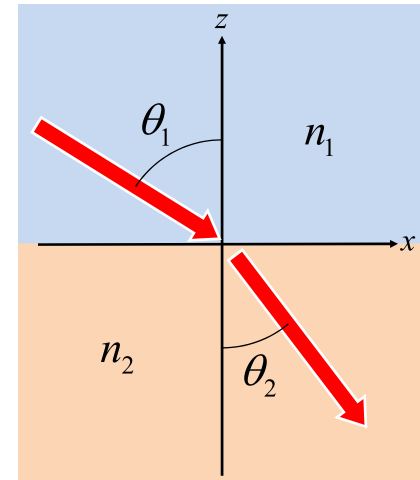

Snell's law quantifies the angles of light rays at an interface.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

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Critical Angle θ_c

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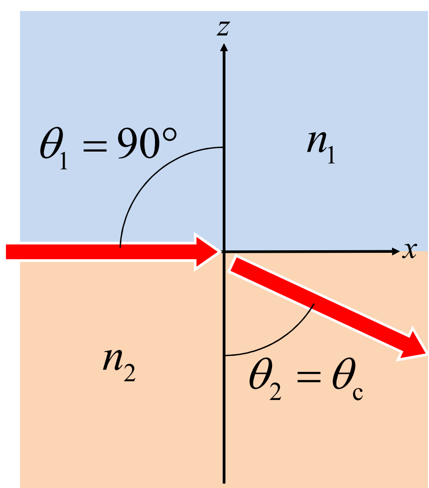

There exists a special angle, the *critical angle*, where the ray in the low-index medium is at 90°.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n_1 \sin 90° = n_2 \sin\theta_c$$
$$n_1 = n_2 \sin\theta_c$$
$$\sin\theta_c = n_1 / n_2$$
$$\theta_c = \sin^{-1}\left(n_1/n_2\right)$$

$$\theta_c = \sin^{-1}\left(n_1/n_2\right)$$
$$\text{where } n_2 > n_1$$

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Total Internal Reflection (TIR)

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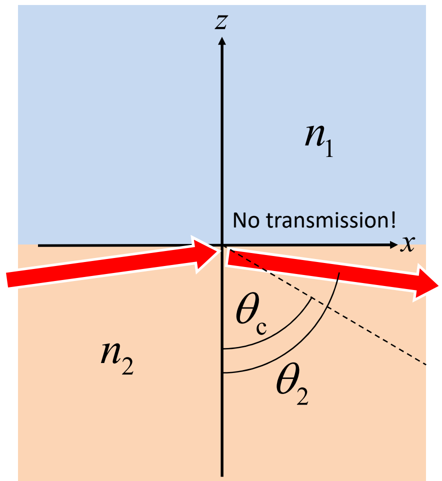

When a light ray is incident onto an interface at an angle greater than the critical angle, the light completely reflects and no light is transmitted.

This is called *total internal reflection* (TIR).

$$\theta_2 > \theta_c$$

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The Slab Waveguide

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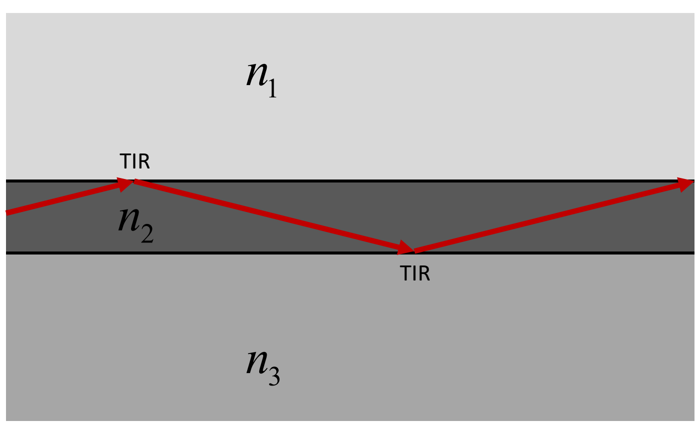
If we "sandwich" a slab of high-index material between two materials with lower refractive index, we form a slab waveguide.

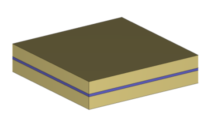


<u>Conditions</u>

$n_2 > n_1$

and

$n_2 > n_3$



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Ray Tracing Picture

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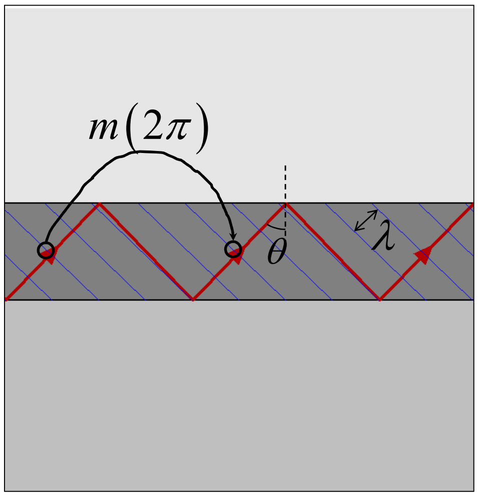

The round trip phase of a ray must be an integer multiple of 2π. Otherwise the wave will interfere with itself and escape from the slab.

Because of this, only certain angles are allowed to propagate in the waveguide.

This is the origin of discrete modes in a waveguide.

$$\beta = k_0 n_{\text{eff}} = k_0 n \sin\theta$$

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Rigorous Analysis

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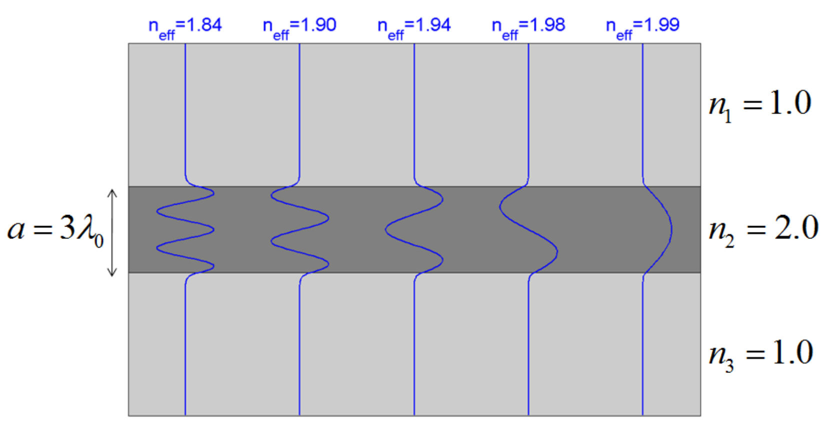

$$\beta = k_0 n_{\text{eff}} = k_0 n \sin\theta$$

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Slab Vs. Channel Waveguides

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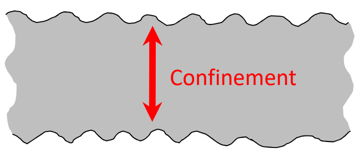
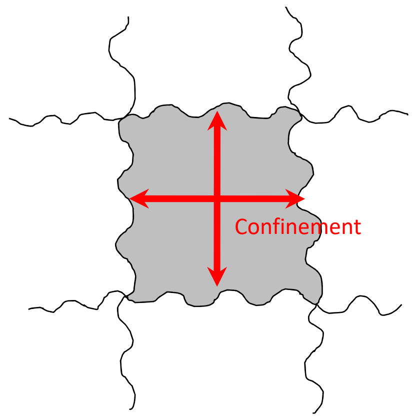
Slab waveguides confine energy in only one transverse direction.



Channel waveguides confine energy in both transverse directions.



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Mathematical Form of Solution

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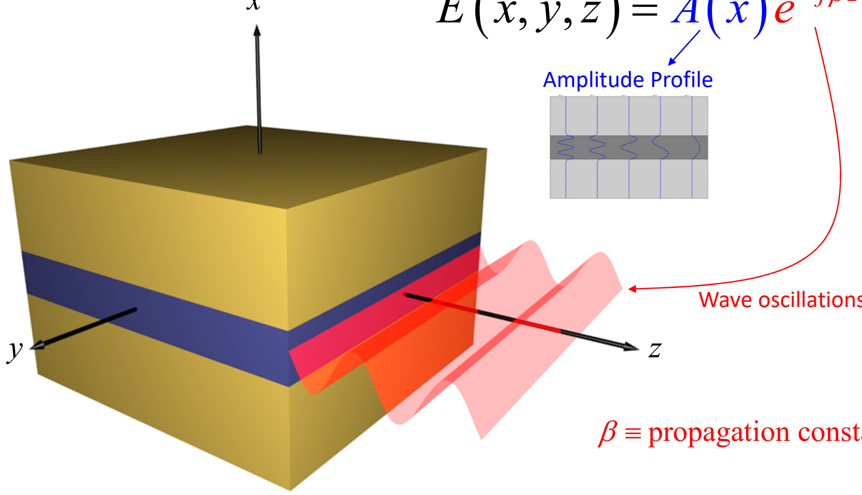
$$\vec{E}(x,y,z) = \vec{A}(x)e^{-j\beta z}$$



$\beta \equiv$ propagation constant

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Computational Methods in EE

Formulation

What is Formulation?

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Formulation is the initial analytical work we do before implementing a computer code.

Usually we start with the governing equation(s) and end with the matrix equation to be solved.

Governing Equations

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$
$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E}$$

Matrix Equation

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$$

Governing Equations

Computational Methods in EE

Since this is an electrodynamics problem, we start with Maxwell's curl equations.

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$
$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E}$$

Vector Curl

The curl of a vector is a measure of the vector field's tendency to circulate about an axis. The curl quantity is directly along this axis and the magnitude measures the strength of the circulation.

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{a}_z$$

Expand Governing Equations (1 of 2)

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If we expand the first equation into its vector components, we get

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{a}_z = -j\omega\mu\left(H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z\right)$$

The vector components on each side must be equal.

$$x\text{-component:}\quad \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$y\text{-component:}\quad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$z\text{-component:}\quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

Expand Governing Equations (2 of 2)

Computational Methods in EE

We now have six equations.

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

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How to Reduce Dimensions

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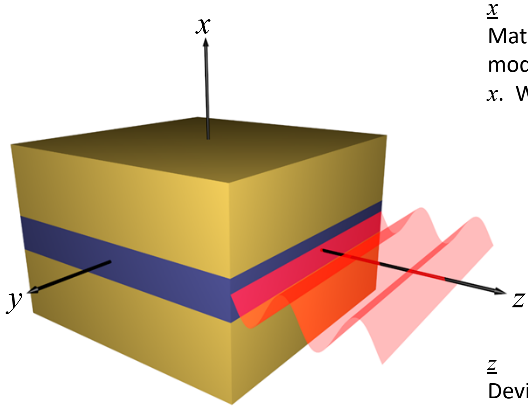
It is always good practice to minimize the number of dimensions utilized in a numerical analysis.



$\underline{x}$
Material changes as a function of x. The mode profile will change as a function of x. We must retain this dimension.

$\underline{y}$
Device is uniform. Wave does not propagate in this direction. Mode profile is uniform.

$$\frac{\partial}{\partial y} = 0$$

$\underline{z}$
Device is uniform. Wave propagates in this direction so wave phase is increasing.

$$\frac{\partial}{\partial z} = -j\beta$$

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Apply $\partial/\partial y = 0$

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Since nothing is changing in the y direction, any derivative with respect to y must be zero.

$$\cancel{\frac{\partial E_z}{\partial y}} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \qquad \rightarrow \qquad -\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \qquad \rightarrow \qquad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \cancel{\frac{\partial E_x}{\partial y}} = -j\omega\mu H_z \qquad \rightarrow \qquad \frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

$$\cancel{\frac{\partial H_z}{\partial y}} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \qquad \rightarrow \qquad -\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \qquad \rightarrow \qquad \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \cancel{\frac{\partial H_x}{\partial y}} = j\omega\varepsilon E_z \qquad \rightarrow \qquad \frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

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Two Distinct Mode Types

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Our revised governing equations have separated into two distinct mode types.

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

We will analyze the Ey mode

Mode Type 1 – Ey Mode

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

Mode Type 2 – Hy Mode

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

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What About $\partial/\partial z$?

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Our guided mode has the following mathematical form

$$\vec{E}(x,y,z)=\vec{A}(x)e^{-j\beta z}$$

Let's calculate the partial derivative with respect to z and see what happens.

$$\frac{\partial}{\partial z}\vec{E}(x,y,z)=\frac{\partial}{\partial z}\left[\vec{A}(x)e^{-j\beta z}\right]=\vec{A}(x)\frac{\partial}{\partial z}e^{-j\beta z}+\cancel{e^{-j\beta z}\frac{\partial}{\partial z}\vec{A}(x)}$$

$$=-j\beta\underbrace{\vec{A}(x)e^{-j\beta z}}_{\vec{E}(x,y,z)}=-j\beta\vec{E}(x,y,z)$$

We conclude that

$$\frac{\partial}{\partial z}=-j\beta$$

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1D Governing Equations

Computational Methods in EE

The equations for the Ey mode were

$$\frac{\partial}{\partial z}H_x-\frac{\partial H_z}{\partial x}=j\omega\varepsilon E_y$$

$$-\frac{\partial}{\partial z}E_y=-j\omega\mu H_x$$

$$\frac{\partial E_y}{\partial x}=-j\omega\mu H_z$$

We simply replace $\partial/\partial z$ with $-j\beta$.

$$-j\beta H_x-\frac{dH_z}{dx}=j\omega\varepsilon E_y$$

$$j\beta E_y=-j\omega\mu H_x$$

$$\frac{dE_y}{dx}=-j\omega\mu H_z$$

The partial derivative has become an ordinary derivative because there is only one independent variable remaining.

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Normalize

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Before converting our equations to matrix form, we should normalize the spatial coordinate x to put it in terms of wavelength in some manner.

$$\tilde{x} \overset{???}{=} \frac{x}{\lambda_0}$$

Additionally, it will be mathematically convenient to normalize by multiplying x by the free space wave number k_0.

$$\tilde{x} = k_0 x \qquad k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega\sqrt{\mu\varepsilon}}{n}$$

Normalizing Maxwell's Eqs.

Computational Methods in EE

We start with the following equation,

$$-j\beta H_x - \frac{dH_z}{dx} = j\omega\varepsilon E_y$$

and replace x with $\tilde{x}/k_0$.

$$-j\beta H_x - k_0\frac{dH_z}{d\tilde{x}} = j\omega\varepsilon E_y$$

Next we divide both sides of the equation by k_0.

$$-j\frac{\beta}{k_0}H_x - \frac{dH_z}{d\tilde{x}} = \frac{j\omega\varepsilon}{k_0}E_y$$

Recognizing that $\beta = k_0 n_{\text{eff}}$, this equation becomes

$$-jn_{\text{eff}}H_x - \frac{dH_z}{d\tilde{x}} = \frac{j\omega\varepsilon}{k_0}E_y$$

$$= \frac{j\omega\varepsilon_0\varepsilon_r}{\omega\sqrt{\mu_0\varepsilon_0}}E_y = j\sqrt{\frac{\varepsilon_0}{\mu_0}}\varepsilon_r E_y$$

Normalized Equations

Computational Methods in EE

Applying our normalizations to all equations, we get

$$-jn_{\text{eff}}H_x - \frac{dH_z}{d\tilde{x}} = j\sqrt{\frac{\varepsilon_0}{\mu_0}}\varepsilon_r E_y$$

$$jn_{\text{eff}}E_y = -j\sqrt{\frac{\mu_0}{\varepsilon_0}}\mu_r H_x$$

$$\frac{dE_y}{d\tilde{x}} = -j\sqrt{\frac{\mu_0}{\varepsilon_0}}\mu_r H_z$$

Last, at optical frequencies, the permeability is negligible so $\mu_r = 1$.

$$-jn_{\text{eff}}H_x - \frac{dH_z}{d\tilde{x}} = j\sqrt{\frac{\varepsilon_0}{\mu_0}}\varepsilon_r E_y$$

$$jn_{\text{eff}}E_y = -j\sqrt{\frac{\mu_0}{\varepsilon_0}}H_x$$

$$\frac{dE_y}{d\tilde{x}} = -j\sqrt{\frac{\mu_0}{\varepsilon_0}}H_z$$

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Final Governing Equation

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We solve the last two equations for H_x and H_z.

$$-jn_{\text{eff}}H_x - \frac{dH_z}{d\tilde{x}} = j\sqrt{\frac{\varepsilon_0}{\mu_0}}\varepsilon_r E_y$$

$$jn_{\text{eff}}E_y = -j\sqrt{\frac{\mu_0}{\varepsilon_0}}H_x \quad \rightarrow \quad H_x = -n_{\text{eff}}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_y$$

$$\frac{dE_y}{d\tilde{x}} = -j\sqrt{\frac{\mu_0}{\varepsilon_0}}H_z \quad \rightarrow \quad H_z = j\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{dE_y}{d\tilde{x}}$$

These are substituted into the first equation to get a single equation containing only E_y. This is why it was called the E_y mode.

$$-jn_{\text{eff}}H_x - \frac{dH_z}{d\tilde{x}} = j\sqrt{\frac{\varepsilon_0}{\mu_0}}\varepsilon_r E_y$$

$$-jn_{\text{eff}}\left(-n_{\text{eff}}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_y\right) - \frac{d}{d\tilde{x}}\left(j\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{dE_y}{d\tilde{x}}\right) = j\sqrt{\frac{\varepsilon_0}{\mu_0}}\varepsilon_r E_y$$

$$n_{\text{eff}}^2 E_y - \frac{d^2E_y}{d\tilde{x}^2} = \varepsilon_r E_y \longrightarrow \frac{d^2E_y}{d\tilde{x}^2} + \varepsilon_r E_y = n_{\text{eff}}^2 E_y$$

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Eigen-Value Problem

Computational Methods in EE

For optical problems, we like to put everything in terms of refractive index. We have $\varepsilon_{\mathrm{r}} = n^2$.

$$\frac{d^2 E_y}{d\tilde{x}^2} + n^2 E_y = n_{\mathrm{eff}}^2 E_y$$

Our governing equation can be rearranged to the form of a standard eigen-value problem.

$$\boxed{\left[\frac{d^2}{d\tilde{x}^2} + n^2(x)\right] E_y(x) = n_{\mathrm{eff}}^2 E_y(x)}$$

Matrix Form

Computational Methods in EE

We go term-by-term to write the equation in matrix form.

$$\left[\frac{d^2}{d\tilde{x}^2} + n^2(x)\right] E_y(x) = n_{\mathrm{eff}}^2 E_y(x)$$

$$\left(\mathbf{D}_{\tilde{x}}^2 + \mathbf{n}^2\right)\mathbf{e}_y = n_{\mathrm{eff}}^2 \mathbf{e}_y$$

or

$$\left(\mathbf{D}_{\tilde{x}}^2 + \boldsymbol{\varepsilon}\right)\mathbf{e}_y = n_{\mathrm{eff}}^2 \mathbf{e}_y$$

Eigen Matrix

Eigen Value

Computational Methods in EE

Solution

Solving the Eigen-Value Problem

Computational Methods in EE

$$\left(\mathbf{D}_{\tilde{x}}^2 + \boldsymbol{\varepsilon}\right)\mathbf{e}_y = n_{\text{eff}}^2 \mathbf{e}_y \quad \rightarrow \quad \begin{array}{l} \mathbf{V} \equiv \text{Eigen-vector matrix} \\ \boldsymbol{\lambda} \equiv \text{Eigen-value matrix} \end{array}$$

$$\mathbf{V} = \left[\begin{bmatrix} e_y^{(1)}(1) \\ e_y^{(1)}(2) \\ e_y^{(1)}(3) \\ \vdots \\ e_y^{(1)}(N_x-1) \\ e_y^{(1)}(N_x) \end{bmatrix} \begin{bmatrix} e_y^{(2)}(1) \\ e_y^{(2)}(2) \\ e_y^{(2)}(3) \\ \vdots \\ e_y^{(2)}(N_x-1) \\ e_y^{(2)}(N_x) \end{bmatrix} \cdots \begin{bmatrix} e_y^{(M)}(1) \\ e_y^{(M)}(2) \\ e_y^{(M)}(3) \\ \vdots \\ e_y^{(M)}(N_x-1) \\ e_y^{(M)}(N_x) \end{bmatrix} \right]$$

M = # modes
Usually $M = N_x$

$$\mathbf{D} = \begin{bmatrix} \left(n_{\text{eff}}^{(1)}\right)^2 & & & \\ & \left(n_{\text{eff}}^{(2)}\right)^2 & & \\ & & \ddots & \\ & & & \left(n_{\text{eff}}^{(M)}\right)^2 \end{bmatrix}$$

Eigen-vectors and eigen-values come in pairs.

Do not mix up their pairing!

Visualizing the Solution

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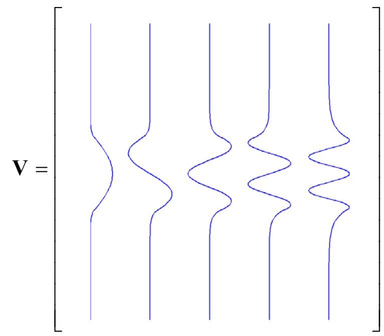
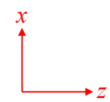
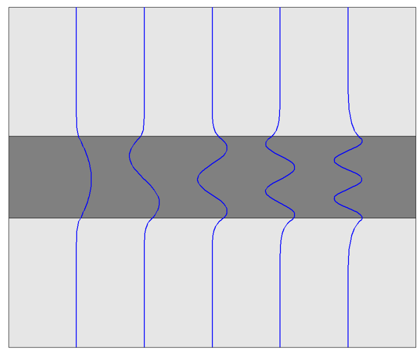
The columns of the eigen-vector matrix are pictures of the modes.







The eigen-values are the effective refractive indices of the modes squared.

Computational Methods in EE

Implementation in MATLAB

Implementation Outline

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1. Initialize MATLAB
2. Dashboard (materials, dimensions, etc.)
3. Calculate Grid
4. Build Device on Grid
5. Perform Finite-Difference Analysis
6. Visualize the Results

Dashboard

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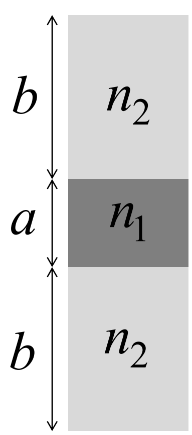

How big should we make b?

→ Enough to allow the mode to decay to zero before reaching the boundary.

What grid resolution should we use?

→ Convergence

```
% slabdemo.m

% INITIALIZE MATLAB
close all;
clc;
clear all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% DASHBOARD
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% FREE SPACE WAVELENGTH
lam0 = 1.0;

% SLAB PARAMETERS
n1 = 2.0;
n2 = 1.0;
a  = 3*lam0;

% GRID
b    = 5*lam0;
NRES = 10;
dx   = lam0/NRES;

% NUMBER OF MODES TO CALCULATE
M = 5;
```

Build Device on Grid

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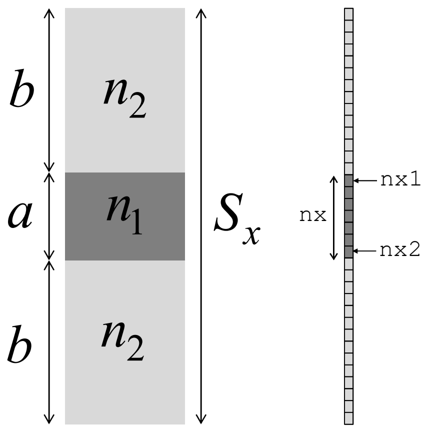


```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% BUILD WAVEGUIDE
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% COMPUTE GRID
Sx = a + 2*b;
Nx = ceil(Sx/dx);
Sx = Nx*dx;

xa = [0.5:Nx-0.5]*dx;
xa = xa - mean(xa);

% COMPUTE START AND STOP INDICES
nx  = round(a/dx);
nx1 = round((Nx - nx)/2);
nx2 = nx1 + nx - 1;

% BUILD N
N            = zeros(Nx,1);
N(1:nx1-1)   = n2;
N(nx1:nx2)   = n1;
N(nx2+1:Nx)  = n2;
```

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Perform Finite-Difference Analysis

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$$\mathbf{D}_{\tilde{x}}^2 = \frac{1}{(k_0 \Delta x)^2} \begin{bmatrix} -2 & 1 & & & & & \\ 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & & \\ & & & \ddots & & & \\ & & & 1 & -2 & 1 & \\ & & & & 1 & -2 & 1 \\ & & & & & 1 & -2 \end{bmatrix}$$

$$\mathbf{N} = \begin{bmatrix} n(1) & & & & \\ & n(2) & & & \\ & & n(3) & & \\ & & & \ddots & \\ & & & & n(N_x) \end{bmatrix}$$

$$\underbrace{\left(\mathbf{D}_{\tilde{x}}^2 + \mathbf{N}^2\right)}_{\mathbf{A}} \mathbf{e}_y = n_{\text{eff}}^2 \mathbf{e}_y$$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% PERFORM FD ANALYSIS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% CALCULATE k0
k0 = 2*pi/lam0;

% BUILD DX2
DX2 = sparse(Nx,Nx);
DX2 = spdiags(+1*ones(Nx,1),-1,DX2);
DX2 = spdiags(-2*ones(Nx,1), 0,DX2);
DX2 = spdiags(+1*ones(Nx,1),+1,DX2);
DX2 = DX2 / (k0*dx)^2;

% MAKE N DIAGONAL
N = diag(sparse(N(:)));

% SOLVE EIGEN-VALUE PROBLEM
A      = DX2 + N^2;
[V,D]  = eig(full(A));
NEFF   = sqrt(diag(D));
```

Topic 7b -- Slab Waveguide Analysis

Slide 36

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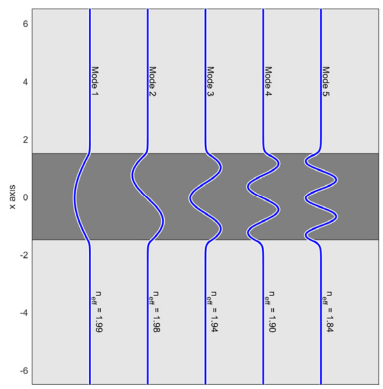
Visualize the Results

Computational Methods in EE



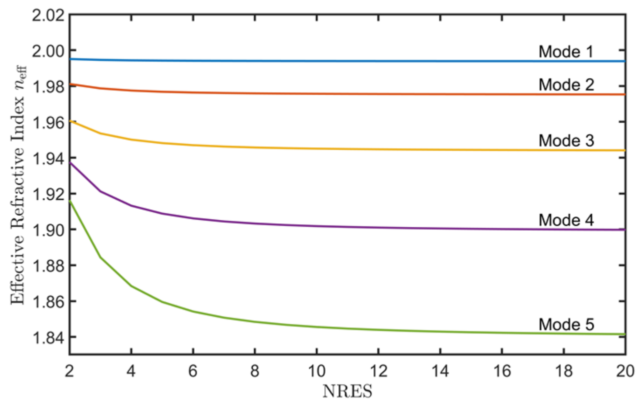

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% VISUALIZE
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% SORT MODES
[~,ind] = sort(real(NEFF),'descend');
V       = V(:,ind);
NEFF    = NEFF(ind);

% OPEN FIGURE WINDOW
figure('Color','w');
hold on;

% DRAW SLAB WAVEGUIDE
x = [0 2*(M+1) 2*(M+1) 0 0];
y = [ -b-a/2 -b-a/2 b+a/2 b+a/2 -b-a/2 ];
fill(x,y,0.9*[1 1 1]);
y = [ -a/2 -a/2 a/2 a/2 -a/2 ];
fill(x,y,0.5*[1 1 1]);

% DRAW AND LABEL MODES
for m = 1 : M
    x0 = 2*m;
    y0 = (a + b)/2;
    x = x0 + 3*V(:,m);
    y = linspace(-b-a/2,b+a/2,Nx);
    line(x,y,'Color','w','LineWidth',4);
    h = line(x,y,'Color','b','LineWidth',2);
    text(x0,y0,['Mode ' num2str(m)],'Rotation',-90,...
         'HorizontalAlignment','center','VerticalAlignment','bottom');
    text(x0,-y0,['n_{eff} = ' num2str(NEFF(m),'%4.2f')],'Rotation',-90,...
         'HorizontalAlignment','center','VerticalAlignment','bottom');
end

% SET GRAPHICS VIEW
hold off;
h2 = get(h,'Parent');
set(h2,'XTick',[]);
axis equal tight;
ylabel('x axis');
```


Computational Methods in EE

More About Resolution and Spacer Regions

Convergence Study for NRES

Computational Methods in EE

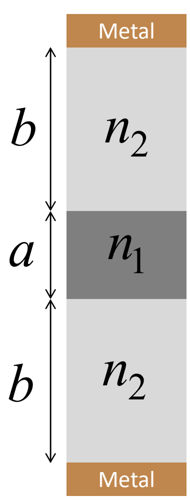


$$\Delta x = \frac{\lambda_0}{\text{NRES}}$$

Notes
- Higher-order modes converge slower.
- Higher-order modes have a smaller n_{eff}.

Spacer Region b

Computational Methods in EE



Remember the Dirichlet boundary conditions? Values outside of the grid are forced to zero.

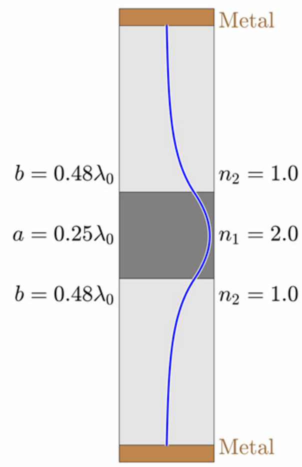
This means we really are simulating a slab waveguide inside of a large metal waveguide.

It is only possible to get an accurate simulation of the slab waveguide when the metal waveguide is large enough.

We must choose b to be large enough to ensure the metal waveguide is insignificant.

Effect of Spacer Region Size

Computational Methods in EE

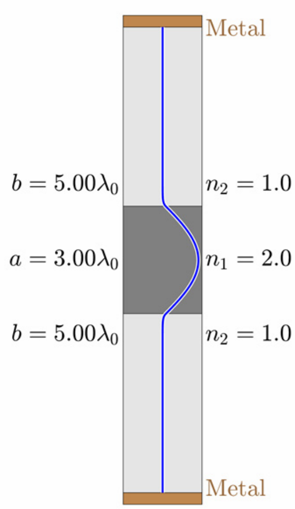


$n_{\text{eff}} = 1.6621$

If the spacer region b is too small, the outer metal waveguide becomes significant and the results for the slab are not accurate.

Conditions for Large Evanescent Fields: *Thin Waveguides*

Computational Methods in EE

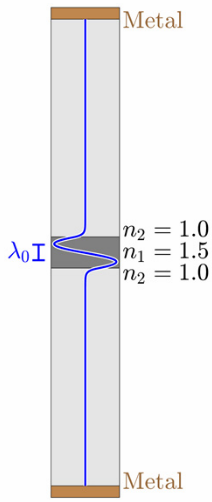


Thin dielectric waveguides have large evanescent fields.

The spacer region b must be big enough to sufficiently encompass the evanescent field in order to give an accurate simulation.

Conditions for Large Evanescent Fields: *Modes Near Cutoff*

Computational Methods in EE

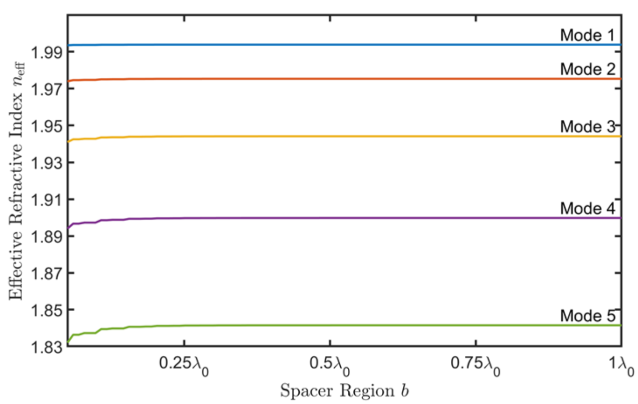


Guided modes operating near cutoff have very large evanescent fields.

The spacer region b must be big enough to sufficiently encompass the evanescent field in order to give an accurate simulation.

Convergence Study for b

Computational Methods in EE



Notes
- Under normal circumstances, the spacer region size can be ~$0.25\lambda_0$.
- Modes near cutoff require larger spacer regions to resolve.
- Thin waveguides may require larger spacer regions.
- Always check for convergence of spacer region size.