Problem #1: Wave Equation

Starting with Maxwell’s curl equations
\[ \nabla \times \vec{E} = -j\omega\mu \vec{H}, \quad (1) \]
\[ \nabla \times \vec{H} = j\omega\varepsilon \vec{E}, \quad (2) \]
derive the wave equation for the electric field in a linear, homogeneous, and isotropic (LHI) medium.
\[ \nabla^2 \vec{E} + (k_0 n)^2 \vec{E} = 0 \quad (3) \]
In this equation, \( n \) is refractive index and \( k_0 \) is the free space wave number. These are defined as
\[ n = \sqrt{\mu/\varepsilon}, \quad (4) \]
\[ k_0 = 2\pi/\lambda_0, \quad (5) \]

Problem #2: Dispersion Relation

Given that the solution to the wave equation is a plane wave
\[ \vec{E}(\vec{r}) = \vec{E}_0 e^{j\vec{k}\cdot\vec{r}}, \quad (6) \]
substitute this solution back into the wave equation to derive the dispersion relation for LHI media given in Eq. (7).
\[ k_x^2 + k_y^2 + k_z^2 = (k_0 n)^2 \quad (7) \]

Problem #3: Inverse of a Block 2x2 Matrix

Using proper matrix algebra rules, derive the following expression for the inverse of a 2x2 block matrix. Remember that \( A, B, C, \) and \( D \) are matrices themselves.
\[ \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} DX^{-1} & -BX^{-1} \\ -CX^{-1} & AX^{-1} \end{bmatrix} \quad X = AD - BC \quad (8) \]

Problem #4: Block Matrix Division

Using proper matrix algebra rules, simplify the following block matrix expression where \( W_i, W_j, V_i \) and \( V_j \) are themselves matrices.
\[ \begin{bmatrix} W_i & W_j \\ V_i & -V_j \end{bmatrix}^{-1} \begin{bmatrix} W_i & W_j \\ V_i & -V_j \end{bmatrix}, \quad (9) \]
Your final answer should be
\[ \frac{1}{2} \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & A_{ij} \end{bmatrix} \quad \text{where} \quad A_{ij} = W_i^{-1}W_j + V_i^{-1}V_j \\
B_{ij} = W_i^{-1}W_j - V_i^{-1}V_j, \quad (10) \]