EE 5337
Computational Electromagnetics (CEM)

Lecture #6
TMM Extras

Outline
• Circuit/Wave Equivalence
Circuit Wave Equivalence

Circuit-Wave Equivalence

Lecture 6
Slide 4
Derivation (1 of 3)

We wish to derive equations that convert between the transmission line framework and the transfer matrix method framework.

\[ \gamma, Z \leftrightarrow \mu_r, \varepsilon_r \]

Step 1 – Extract the complex refractive index \( n \) and complex impedance \( \eta \) from the complex propagation constant \( \gamma \) and characteristic impedance \( Z \).

\[
e^{-jknz} = e^{-\gamma z}
\]

\[
\downarrow
\]

\[
n = \frac{\gamma}{jk_0}
\]

Derivation (2 of 3)

Step 2 – Relate the complex refractive index \( n \) and complex impedance \( \eta \) to the complex permittivity \( \varepsilon_r \) and complex permeability \( \mu_r \).

\[
n = \sqrt{\mu_r \varepsilon_r}
\]

\[
\eta = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}
\]

Step 3 – Solve the above equations for the complex permittivity \( \varepsilon_r \) and complex permeability \( \mu_r \).

\[
\varepsilon_r = n \frac{\eta_0}{\eta}
\]

\[
\mu_r = n \frac{\eta}{\eta_0}
\]

Step 4 – Replace the complex refractive index \( n \) and complex impedance \( \eta \) with the complex propagation constant \( \gamma \) and characteristic impedance \( Z \) from Step 1.

\[
\varepsilon_r = \frac{\gamma}{j\omega \varepsilon_0 Z}
\]

\[
\mu_r = \frac{\gamma Z}{j\omega \mu_0}
\]
Step 5 – Solve the equations from Step 4 for the complex propagation constant $\gamma$ and characteristic impedance $Z$.

\[
\gamma = jk_0 \sqrt{\mu \varepsilon_r} \\
Z = \eta_0 \sqrt{\frac{\mu}{\varepsilon_r}}
\]