

```
GRID SIZE: Nx = 1, Ny = 3
RESOLUTION: dx = 1, dy = 2
BC's: DIRICHLET
DX =
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
D2X =
[ 0 0 0 ]
[ 0 0 0 ]
DY =
[ 0 0.25 0 ]
[ -0.25 0 0.25 ]
[ 0 -0.25 0 ]
D2Y =
[ -0.5 0.25 0 ]
[ 0.25 -0.5 0.25 ]
[ 0 0.25 -0.5 ]
GRID SIZE: Nx = 1, Ny = 5
RESOLUTION: dx = 1, dy = 2
BC's: DIRICHLET
DX =
D2X =
 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} 
DY =
    \begin{array}{cccccc} 0 & 0.25 & 0 \\ -0.25 & 0 & 0.25 \\ 0 & -0.25 & 0 \\ 0 & 0 & -0.25 \\ 0 & 0 & 0 \end{array}
                                                           0
0
0
0.25
 [ -0.25
                                           0.25
0
-0.25
D2Y =
 \begin{bmatrix} -0.5 & 0.25 & 0 & 0 & 0 & 0 \\ 0.25 & -0.5 & 0.25 & 0 & 0 & 0 \\ \end{bmatrix} \\ \begin{bmatrix} 0.25 & -0.5 & 0.25 & 0 \\ 0 & 0.25 & -0.5 & 0.25 \\ \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0.25 & -0.5 & 0.25 \\ 0 & 0 & 0.25 & -0.5 \\ \end{bmatrix}
```

```
GRID SIZE: Nx = 3, Ny = 1
RESOLUTION: dx = 1, dy = 2
BC's: DIRICHLET

DX =

[ 0 0.5 0 ]
[ -0.5 0 0.5 ]
[ 0 -0.5 0 ]

D2X =

[ -2 1 0 ]
[ 1 -2 1 ]
[ 0 1 -2 ]
```



```
DA =
[ 0 0 0 ]
[ 0 0 0 ]
D2Y =
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
GRID SIZE: Nx = 5, Ny = 1
RESOLUTION: dx = 1, dy = 2
BC's: DIRICHLET
DX =
 \begin{bmatrix} & 0 & 0.5 & 0 & 0 \\ I & -0.5 & 0 & 0.5 & 0 \\ I & 0 & -0.5 & 0 & 0.5 \\ I & 0 & 0 & -0.5 & 0 \\ I & 0 & 0 & 0 & -0.5 \end{bmatrix} 
                                                          0 ]
0 ]
0 ]
0.5 ]
D2X =
            1 0 0 0 0 0 -2 1 0 0 0 1 -2 1 0 0 0 0 1 -2 1 0 0 0 1 -2
[ -2
[ 1
[ 0
[ 0
DY =
D2Y =
GRID SIZE: Nx = 3, Ny = 3
RESOLUTION: dx = 1, dy = 2
BC's: DIRICHLET
DX =
                                                       0
0
0.5
0
-0.5
                 0.5
0
-0.5
0
0
0
                                 0
0.5
0
0
                                                                                                  0
0
0
0
0
0
0.5
0
                                                                                                                      0
0
0
                                                                              0
    -0.5
0
0
                                            0
0
0
-0.5
                                                                                            0 0 0
                                                                             0
                                                                          0.5
                                                  0 0
                                                                                     0 0 -0.5
          0 0
                                                                              0 0
                                                                                                                      0
                        0
D2X =
                                                                             0
0
0
                      0
1
-2
0
0
0
0
0
                               0
0
0
-2
1
0
                                         0
0
0
                                                 0
0
0
0
1
-2
0
0
    -2
1
0
0
0
0
0
0
                                                           0
0
0
0
0
                                                                   0
0
0
0
0
0
1
-2
                                       -2
1
0
0
                                                                             0
                                0 0
```



```
DY =
                                                                                        0
0
0.25
0
0
0
0
                                                                                                             0
0
0
0.25
0
0
              0
                                                                                                                                               0
0
0
0
0
0
0.25
0
                     0
0
-0.25
0
                                                                                                                              0
0
0.25
0
0
0
   -0.25
0
0
                                                      -0.25
0
                                      -0.25
0
0
D2Y =
                               0
0
-0.5
0
0
0.25
0
                                                            0
0.25
0
0
-0.5
0
0
                                                                                                         0
0
0
0
0.25
0
0
                  0
-0.5
0
0
0.25
0
                                                                                           0
0
0
0.25
0
                                                                                                                                0
0
0
0
                                                                             00.25
                                                0 0 0 -0.5
                                              0
0
0.25
0
            0
                                                                                            -0.5
0
0
GRID SIZE: Nx = 3, Ny = 5
RESOLUTION: dx = 1, dy = 2
BC's: DIRICHLET
DX =
                                    0
0.5
0
0
0
0
                                                                                            0
0
0
0
0
0
0.5
0
0
-0.5
0
0
                                                                                                                                                                        0
0
-0.5
0
                                                              0.5
0.5
0
-0.5
0
0
0
0
0
                                                                                                                                        0
0
0
0
0
0
0
0
0
0
                                                                                                                                                              0 0 0 0 0 0
            0
                          0 0 0
                                                                                0.5
                                                                                                                            0.5
                                                       0
            0 0 0 0 0 0
                          0
0
0
0
0
                                                                                                                                                       0.5
0
-0.5
0
                                                       0 0 0
                                                                                                                                0
                                                                                     0
D2X =
                                                      0
                                                                0
                                                                                    0
                                   0
                                             0
     -2
1
0
0
0
0
0
0
0
0
0
0
0
                          0
1
                                                                          0
                                                                                              0 0 0 0 0 0 0
                                                                                                       0
                                                                                                               0
0
0
0
0
0
0
0
0
0
1
-2
0
0
                                                                                                                         0
0
0
0
0
0
0
0
0
0
0
                                                                                                                                   0
                                                                        0
0
0
0
1
-2
1
0
0
0
                                  0
-2
1
0
0
0
0
0
0
0
0
                                                                                     0
0
0
0
                                           -2
1
0
0
0
0
0
0
0
                                                               0
-2
1
0
                                                       0 0 0 0 0 0 0
                                                                                  1
-2
0
0
0
0
                                                                                            -2
1
0
0
0
                                                                                                     1
-2
1
0
0
                                                                                                                                            0
0
0
0
1
-2
DY =
                                                                                         0
0.25
0
0
0
0
0
                                                          0.25
0
0
              0
                               0 0 0
                                                                                                                    0
     0
0
-0.25
                                                                                                            0
0
0.25
                                                                                                                            0
0
0
0.25
0
0
0
0
                                                                                                                                                                                 0
0
0
0
0
0
0
0.25
0
0
                                                                                                                                                                                                                                     0
0
0
0
0
0
0
0
0
0
0
0
0
0
                                                                                                                                               0
0
0.25
0
0
                                                0
                     -0.25
0
0
0
                                                                                                                                                                                                  0
              0
0
0
0
0
0
0
                                      -0.25
0
0
                                                                                                                                                                0.25
                                                                        -0.25
0
0
                                                                0
                                                                                                                     0
                                                                                                                                                                                                                   0.25
0
0
0
                                                                                                   0
                                                                                                          -0.25
0
                                                                                                                                                       0
                                                                                                                                                                                                                                                            0
                                                                                                   0 0
                                                                                                                                     0 0
                                                                                                                                                             -0.25
0
                                                                                                                                                      0
                                                                                                                                                                                                                                                            0
```



| DY = [| [0 1 -2 [0 0 0 1 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 | GRID SIZE: RESOLUTION: BC's: DX = [| [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 |
|---|--|---|--|
| 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 1 1 0 1 1 -2 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | dx = 1, dy DIRICHLET 0 0 0.5 0 0.5 0 0 0.7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 -0.5 0 -0.5 0 0 -0.6 0 0.25 0 0.25 0 0 0.2 |
| 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | | = 2 0 | 0 -0.5 0 0 -0.5 25 0 0 0 0.25 0 0 0.25 0 0 0.25 |
| 0.25 0 0.25 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0.25 0 0 0.25 0 0 0 0.25 0 0 0 0.05 0 0 0 0.05 0 0 0 0.05 0 0 0.00 0 0 0.00 |
| 0 0.25 0 0.25 0 0 0.0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0.5 0 0 0.5 -0.5 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 |
| 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | | | 0 0 0 0 0 0 0 0 0 0 0 0 0.25 0 0 0.25 0 0 -0.5 0 |
| 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0.25 0 0 |
| 0 0 0 0 0 0 0 0 0.25 0 0 | | | 0] |
| 0 0 1 0 0 0 1 25 0 1 0 0 0.25 1 0 0 0 1 0 0 0 1 0 0 0 1 | | | |



```
D2Y =
                                  0
0
-0.5
0
                   -0.5
0
                                                                            0.25
0
0
                                                       0 0
                                                               0
0
0
0
-0.5
0
0
0
                                                                                            0
0
0
0.25
0
0
0
0
-0.5
                                                                                                                                        0
0
0
0
0.25
0
0
0
0
-0.5
0
0
                                                                                                                                                                     0
0
0
0
0
0
0
0
0.25
0
                                                                                                                                                                                                                  0
0
0
0
0
0
                   0
0
0
0
                                                -0.5
0
0
0
            0
                                                                                                                                                       0.25
                                 0
0
0.25
0
    0.25
0
0
0
                                                                             -0.5
0
0
                                                                                                                                                                                     0.25
                          0 0
                                               0.25
                                                                            0.25
0
0
0
                                                                                                                         0
0
0
0.25
                                                                                                                                                       -0.5
0
0
                                                                                                                                                                     0
-0.5
0
0
                                         0
                          0 0 0
                                                        0
0
0
0
                                                                      0 0 0
                                         0
GRID SIZE: Nx = 4, Ny = 4
RESOLUTION: dx = 1, dy = 2
BC's: DIRICHLET
DX =
                                                               0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
                                  0.5
                                                  0.5
                                                                                             0
0
0.5
0
-0.5
0
0
0
0
0
                                                                                                              0
0
0
0
0
0
0.5
0
-0.5
                                                                                                                                                        0
0
0
0
0
0
0
0
0
0
0
0
0
                                                                                                                                                                         0
0
0
0
0
0
0
0
0
0
0
                          0
                                                                              0.5
0
-0.5
0
                          0
                                         0
                                                                                                                                 0
                          0
                                                       0
                                                                                                                          -0.5
0
0
            0
0
0
0
                          0
0
0
0
                                                                                                                                                                                                   0.5
0
-0.5
0
                                                                                                                                 0
            0
                                                                                                                                 0
D2X
                                                                                     0
                                             0
                                                                                                                0
0
0
0
0
0
0
0
0
0
1
-2
0
0
0
       1
0
0
0
0
0
0
0
0
0
0
0
0
                                  0
1
-2
0
0
0
0
0
0
0
0
0
0
0
0
0
                                            0
0
0
-2
1
0
0
0
0
0
0
0
0
0
                                                                         0
0
0
0
1
-2
0
0
0
0
0
                                                                                                                          0
0
0
0
0
0
0
0
0
0
0
0
0
                                                                                     0 0 0
                                                               0
0
1
-2
1
0
0
                                                                                  -2
1
0
0
                                                                                               0
0
0
0
                                                                                                        1
0
0
0
                                                       0 0
                                                                                     0 0
                                                                                                                                   -2
1
0
                                                                 0 0
DY =
                                                                          0.25
0
0
0
                                               0
                                                                0
0
0
              0
                              0
                                                                                             0
0.25
                                                                                                                     0
                                                                                                                            0
0
0.25
0
0
0
0
0
0
0
0
0
0
0
                                                                                                                                                      0
                                                                                                                                                                                                                                      0
                      0
0
0
0
-0.25
0
                                                                                                             0.25
0
0
                                                                                                                                                0
0
0.25
                                                                                                   0 0
               0
                                                                                                                                                                                                                    0
0
0
0
0
0
0
0.25
0
0
              0
                                                                 0
                                                                                                   0 0 0
                                                                                                                                                       0
0
0
0
0
                                                                                                                                                                                  0
0.25
0
0
0
0
0
0
                                                                                                                     0
0
0
                                                                                                                                                                                                                                                              0
0
0
                                                                                                                                                                                                  0.25
0
0
0
                                                        -0.25
0
0
              0
                                                0
0
0
                                                                                         -0.25
0
0
0
0
               -0.25
0
0
                                                                                                                                                                                                                                                      0.25
0
0
                                                                                                                                             -0.25
0
0
                                                                                                                                                                         0
```



```
D2Y =
                               0.25
0
0
                                                    0
0
0
0.25
  -0.5
0
         0
-0.5
                           0
                                      0
0.25
                                                                                                                   0
                              0
             0
                        -0.5
                                                 0
  0.25
                0.25
         0.25
                           0
                                   0
                                      -0.5
                                                 0
                                                        0
                                                                0
                                                                           0.25
             0
                           0
                                                    -0.5
                    0
                        0.25
                                                 0
                                                                                  0.25
                        0 0.25
0 0
                                                 0
                                                            -0.5
                                                                                         0.25
                                                        0
                                                                               0
                                                                                      0
                                      0.25
                                         0 0.25
0 0
0 0
             Ω
                           0
                                   0
                                                    0
0.25
                                                                0
                                                                       0
                                                                          -0.5
                                                                                      0
                                                                                             0
                                                                                                    0
                                                                                                       0.25
             0
                                                                                  -0.5
                                                                               0
                                                                          0
                                                                  0.25
                                                            0.25
             0
                                   0
                                                         0
                                                                                          -0.5
                                                                                                -0.5
             0
                           0
                                                                0
                                                                                      0
                                                                                             0
                                   0
                                                                       0 0.25
                                                                              0 0.25
                                                                                                               -0.5
                    0
                           0
                                                 0
                                                                       0
                                                                                                    0
                                          0
GRID SIZE: Nx = 1, Ny = 3
RESOLUTION: dx = 1, dy = 2
BC's: PERIODIC
DX =
[ 0 0 0 ]
D2X =
DY =
[ 0 0.25 -0.25 ]
[ -0.25 0 0.25 ]
[ 0.25 -0.25 0 ]
D2Y =
[ -0.5  0.25  0.25  ]
[ 0.25  -0.5  0.25  ]
[ 0.25  0.25  -0.5 ]
GRID SIZE: Nx = 1, Ny = 5
RESOLUTION: dx = 1, dy = 2
BC's: PERIODIC
DX =
D2X =
[ 0
     0 0 0 0 ]
]
DY =
           0.25 0
0 0.25
                                0 -0.25
0 0
[ -0.25
         -0.25 0
0 -0.25
0 0
                           0.25 0
0 0.25
-0.25 0
0 ]
D2Y =
 \begin{bmatrix} -0.5 & 0.25 & 0 & 0 & 0.25 \\ 0.25 & -0.5 & 0.25 & 0 & 0 \\ 0 & 0.25 & -0.5 & 0.25 & 0 \\ 0 & 0 & 0.25 & -0.5 & 0.25 \\ 0.25 & 0 & 0 & 0.25 & -0.5 \\ \end{bmatrix} 
                           0 0.25 ]
```

```
GRID SIZE: Nx = 3, Ny = 1
RESOLUTION: dx = 1, dy = 2
BC's: PERIODIC
DX =
[ 0 0.5 -0.5 ]
[ -0.5 0 0.5 ]
[ 0.5 -0.5 0 ]
D2X =
[ -2 1 1 ]
[ 1 -2 1 ]
[ 1 1 -2 ]
DY =
[ 0 0 0 ]
[ 0 0 0 ]
D2Y =
[ 0 0 0 ]
[ 0 0 0 ]
GRID SIZE: Nx = 5, Ny = 1
RESOLUTION: dx = 1, dy = 2
BC's: PERIODIC
 \begin{bmatrix} & 0 & 0.5 & 0 & 0 & -0.5 & ] \\ [-0.5 & 0 & 0.5 & 0 & 0 & ] \\ [ & 0 & -0.5 & 0 & 0.5 & 0 & ] \\ [ & 0 & 0 & -0.5 & 0 & 0.5 & ] \\ [ & 0.5 & 0 & 0 & -0.5 & 0 & ] \\ \end{bmatrix} 
D2X =
[ -2 1 0 0 1 ]
[ 1 -2 1 0 0 ]
[ 0 1 -2 1 0 0 ]
[ 0 0 1 -2 1 0 ]
[ 1 0 0 1 -2 1 ]
DY =
D2Y =
GRID SIZE: Nx = 3, Ny = 3
RESOLUTION: dx = 1, dy = 2
BC's: PERIODIC
DX =
                                                   0
0
0
0.5
                           -0.5
0.5
0
                                                                                         0
0
0
                                                                                                     0 ]
0 ]
0 ]
[ -0.5
                                        0
              0-0.5
                                                                  0
                                                                              0
                               0 0
0 -0.5
                                                            -0.5
0.5
                    0
         0
                                                                              0
                                                                                                     Ω
                              0 0.5 -0.5
                                                                                   0 0
0.5 -0.5
0 0.5
-0.5
                   0 0
                                                              0 0
0 0
0 -0.5
                                                                Ő
         0
          0
                                       0
0
0
                                                   0 0
                                0
                    0
                                                                          0.5
```



0 0 0 0 0 0 0 0 0 0 0 0.5 0 -0.5 0 0 0 0 0 0 0 0 -0.5 0.5 0 -0.5 0 0 0 0 0 0.5 0 0 0 0 0 0 0 0 0 0.5 0 -0.5 0 0 0 0 0.5 0 0 0 0 0 0 0 0 0 0

D2X = 0 0 0 0 0 0 1 -2 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 -2 0 0 -2 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -2 1 0 0 0 0 0 0 0 1 1 -2 0 0 0 0 0 0 -2 1 0 0 0 1 -2 0 0 0 0 0 0 0 0



| [0 [0 -0.25 [0 -0.2 [0 -0. | 0 0 0 0 0 0 0 0 0 05 0 0 0 0 -0.25 0 0 0 -0.25 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | -0.25 0 0] 0 -0.25 0] 0 0 -0.25 0] 0 0 0 0 0] 0 0 0 0] 0 0 0 0] 0 0 0 0 |
|--|--|---|---|---|
| [0.25 | 0 -0.5 0 (0 0 -0.5 0 (0.25 0 0 -0.5 0 0 0.25 0 0 0 0 0.25 (0 0 0 0.25 0 | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 1 0 1 0 1 0 1 0 1 0 1 |
| | 0 0 -0.5 0 0.5 0 0 0 0 0.5 0 0 -0.5 0 0.5 0 0 -0.5 0 0.5 0 | 0 0.5 0 0 0 -0.5 0 0.5 0 0 0 -0.5 0 0.5 0 0 -0.5 0 0.5 0 0 0 -0.5 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | | |



| DY = [[[[[[[[[[[[[[[[[[| 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 225 0 0 0 0 0 225 0 0 0 0 0 | 0 0 0 0 0 0 25 0 0 0 25 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 25 0 | 0 0 0 0 0 0 0 | 0 0 0.25 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | -0.2 -0.2 0 0 0.25 | 25 0 0.0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 25 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 25 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0.25 0 0 0 0 0 0 0 0 | 0 0 0 0 0.25 0 0 0 0 0 0 0 0 | -0.25 0 0 0 0.25 0 0 0 0 0 | -0.25 0 0 0 0 0.25 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | |
|---|--|--|---|---|---|---|---|---|--|---|---|---|--|---|---|---------------------------------------|--|
| [| 0 0.25 0 0 0 0 0 0 0 0 0.5 0 0.25 0 0 0 0 0 0 0 | 0.25 0 0 0 0 0 0.25 0 0 | 0 0.25 0 0 0 0 0.25 0 | 0.25 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | -0.5 0 0 0 0.25 0 0 | 0 -0.5 0 0 0 0.25 0 | 0 0 -0.5 0 0 | 0 0 0 -0.5 0 0 0 0.25 | 0 0 0 0 -0.5 | 0.25 0 0 0 0 -0.5 0 0 | 0 0.25 0 0 0 0 -0.5 0 | 0 0.25 0 0 0 | 0 0 0 0.25 0 0 |] | | |
| 1.0e | 0 -0.5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0.5 | 0 0.5 0 0 | 0.5 0.5 0.5 0.5 | 0 -0.5 0 0 0 0 0 0 0 0 0 | 0 0 0.5 0 -0.5 0 0 0 | 0 0 0 0 0.5 0 0.5 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 -0.5 0 0 0.5 | 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 | | |
| D2X = [-2 [1 [0 [1 [0 [0 [0 [0 [0 [0 [0 | 1 0 0 -2 1 1 1 -2 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 1 0 1 -2 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 1 -2 1 0 0 0 0 | 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 1 -2 1 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0] 0] 0] 0] 0] 0] 0] 0] 0] 0] | | | | | | | |

| DY = |
|--|
| [0 0 0 0 0 0.25 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 |
| [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 |
| D2Y = |
| $ \begin{bmatrix} -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 \\ [& 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 \\ [& 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ [& 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ [& 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 \\ [& 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 \\ [& 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 \\ [& 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 \\ [& 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 \\ [& 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ [& 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 & 0 \\ [& 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0.25 \\ [& 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 \\ [& 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 \\ [& 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 \\ [& 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 \\ [& 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 \\ [& 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & -0.5 & 0 \\ [& 0 & 0 & 0$ |
| ************************************** |
| GRID SIZE: Nx = 1, Ny = 3 RESOLUTION: dx = 1, dy = 2 BC's: NEUMANN |
| DX = |
| |
| D2X = |
| |
| DY = |
| [-0.5 |
| D2Y = |
| [0 0 0] [0.25 -0.5 0.25] [0 0 0] |
| GRID SIZE: Nx = 1, Ny = 5 RESOLUTION: dx = 1, dy = 2 BC's: NEUMANN |
| DX = |
| [0 0 0 0 0 0] [0 0 0 0 0 0] |

Benchmarking Document for FDDER



```
D2X =
[ 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 ]
DY =
 \begin{bmatrix} -0.5 & 0.5 & 0 & 0 \\ -0.25 & 0 & 0.25 & 0 \\ 0 & -0.25 & 0 & 0.25 \\ 0 & 0 & -0.25 & 0 \\ 0 & 0 & 0 & -0.5 \end{bmatrix} 
                                                                      0 0
0.25 0
0 0.25
-0.5 0.5
D2Y =
 \begin{bmatrix} & 0 & & 0 & & 0 & & 0 & & 0 & \\ [ & 0.25 & -0.5 & & 0.25 & & 0 & & 0 & ] \\ [ & 0 & 0.25 & -0.5 & & 0.25 & & 0 & ] \\ [ & 0 & 0 & 0.25 & -0.5 & & 0.25 & ] \\ [ & 0 & 0 & 0 & 0 & & 0 & & 0 & ] \\ \end{bmatrix} 
GRID SIZE: Nx = 3, Ny = 1
RESOLUTION: dx = 1, dy = 2
BC's: NEUMANN
                            1 0 ]
0 0.5 ]
-1 1 ]
 [ -0.5
D2X =
[ 0 0 0 ]
[ 1 -2 1 ]
[ 0 0 0 ]
DY =
[ 0 0 0 ]
[ 0 0 0 ]
D2Y =
[ 0 0 0 ]
[ 0 0 0 ]
GRID SIZE: Nx = 5, Ny = 1
RESOLUTION: dx = 1, dy = 2
BC's: NEUMANN
 \begin{bmatrix} & -1 & 1 & 0 \\ I & -0.5 & 0 & 0.5 \\ I & 0 & -0.5 & 0 \\ I & 0 & 0 & -0.5 \\ I & 0 & 0 & 0 \\ \end{bmatrix} 
                                                          0 0 ]
0 0 ]
0.5 0 ]
0 0.5 ]
-1 1 ]
D2X =
[ 0
[ 1
[ 0
[ 0
[ 0
                0 0 0 0 0 ]
-2 1 0 0 ]
1 -2 1 0 0 ]
0 1 -2 1 1 ]
0 0 0 0 0 ]
DY =
```



```
D2Y =
          0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0
0 ]
0 ]
0 ]
0 ]
GRID SIZE: Nx = 3, Ny = 3
RESOLUTION: dx = 1, dy = 2
            BC's: NEUMANN
DX =
                                   0
0.5
1
0
0
0
                                                                                          0
0
0
0
0
0
-1
-0.5
                                                      0 0
                                                                   0
0
0
1
0
-1
0
0
                                                                                                                              0
0
0
                        1
0
-1
0
0
0
0
                                                                                   0 0
                                                                                                                0
0
0
0
0
1
   -0.5
0
                                               -1
-0.5
0
0
0
           0
                                                                                   0
                                                                              0.5
            0
                                                                                   1
                                                                                                                         0.5
D2X
                0
                         0
                                   Ω
                                            0
                                                      0
                                                               0
                                                                                   0 0 0
      0
0
0
0
0
0
                         1
0
0
                                            0 0
                                   0
0
0
1
0
0
0
                                           -2
0
0
0
                                                                                   0
                         0 0 0 0
                                                      1
0
0
0
DY =
                     0
-0.5
0
0
-0.25
0
                                        0
0
-0.5
0
                                                                            0
0.5
0
0
                                                                                                                            0
0
0
0
0.25
                                                           0.5
                                                                                          0
0.5
0
0
0
0
0
       -0.5
                                                                                                           0
0
0
0.25
0
0
0.5
0
                                                                                                                                             0
0
0
0
0
0.25
0
0
             0
                                                         0
0
0
-0.5
0
    -0.25
                                      -0.25
0
0
                                                                                                                              0
             0
                                                                          -0.5
D2Y =
                                                             0
0
0
0
-0.5
0
0
                                                                                                         0
0
0
0
0.25
0
0
                                               0
0
0
-0.5
0
                                                                                                                       0
0
0
0
0
                                                                                          0
0
0
0.25
            0
                                 0
0
0
0
0
0
0.25
0
                                                                                   0
                         0 0 0
            0
                                                                            0 0 0 -0.5 0 0
                  0.25
0
0
0
           0
                                                                                                 0 0 0
                                                      0
                                                                                                                              0
GRID SIZE: Nx = 3, Ny = 5
RESOLUTION: dx = 1, dy = 2
BC's: NEUMANN
DX =
                                                                               0
0
0
0
0.5
                                   0
0.5
1
0
0
0
                                                                   0
0
0
1
0
-1
0
0
0
0
                        1
0
-1
0
0
0
0
0
                                                       0
                                                                                                              0
0
0
0
0
0
1
0
-1
0
0
0
                                                                                                                                                                                                                     0 0 0 0 0 0 0 0
                                                0
0
-1
-0.5
0
0
0
                                                                                           0
0
0
0
0
0
-1
-0.5
0
0
0
                                                                                                                                                                     0
0
0
0
0
0
0
0
0
0
0
0
0
0
     -0.5
                                                                                                                              0
                                                                                                                                                         0
0
0
0
0
0
0
0
            0
                                                                                                                                            0
0
0
0
0
            0
                                                                                   0 0
                                                                                                                         0.5
                                        0
                                                                                                                                                                                                                0
0
0
0
0
0.5
                                                                                                                                      -1
-0.5
0
            0
0
0
0
                         0
0
0
0
                                                                                                                              0
                                                       0
                                                                     0
                                                                                                                              0
                                                                                                                                             0
                                                                                   0
                                                                                                                                                           0
```



| [1 -2 | 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 | 0 0 1 0 0 1 | | | |
|--|--|---|---------------------------------------|--|--|--|--|
| [0 - [0 [-0.25 | 0.5 0 0 -0.5 0 0 .25 0 0 -0.25 | 0.5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 |
| [0 [0.25 [0 0.25 [| 0 0 0 0 0 0 0 0 0 0 0 0 5 0 0 0 0 0.25 0 0 0 0.25 0 0 0 0.25 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 | |
| RESOLUTION: BC's: DX = [| Nx = 5, Ny = dx = 1, dy = NEUMANN 1 | | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 | 0 | |



| D2X = | | | |
|--|---|---|--|
| $ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0$ | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | | |
| DY = | | | |
| [0 -0.5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0.5 0 0 0.5 0 0 0 0.5 0 0 0 0 0 0 0 | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | 0 0 1 0 0 25 0 0 1 0 0 25 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 0 1 |
| D2Y = | | | |
| [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0] 0 0 0 0 0 0] 0 0 0 0 0 | |
| GRID SIZE: Nx = 4, Ny = 4 RESOLUTION: dx = 1, dy = 2 BC's: NEUMANN | | | |
| DX = | | | |
| $ \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ [-0.5] & 0 & 0.5 & 0 & 0 \\ [-0.5] & 0 & 0.5 & 0 & 0 \\ [-0.5] & 0 & 0.5 & 0 & 0 \\ [-0.5] & 0 & 0.5 & 0 & 0 \\ [-0.5] & 0 & 0.5 & 0 & 0 \\ [-0.5] & 0 & 0 & 0 & 0 & -1 \\ [-0.5] & 0 & 0 & 0 & 0 & -1 \\ [-0.5] & 0 & 0 & 0 & 0 & 0 & 0 \\ [-0.5] & 0 & 0 & 0 & 0 & 0 \\ [-0.5] & 0 & 0 & 0 & 0 $ | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0] 0] 0] 0] 0] 0] 0] 0] 0] 0] |



| D2X = | | | | | | | | | | | | | | | | | | | | |
|---|--|--|---|--|---|---|---|---|---|---|---|---|--|--|---|---|---|---|---|---|
| | 0 -2 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 1 -2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 -2 1 0 0 0 0 0 | 0 0 0 0 0 1 -2 0 0 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 | | | | | | | |
| DA = | | | | | | | | | | | | | | | | | | | | |
| [-0 [[-0 . []]]]] [-0 . []]] [[-0 . []]]] [-0 . []] [-0 . []]] [-0 . [] [-0 . []] [-0 . []] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . [] [-0 . []] [-0 . [] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . [] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . [] [-0 . []] [-0 . []] [-0 . [] [-0 . []] [-0 . [] [-0 . []] [-0 . [] [-0 . []] [-0 . [] [-0 . []] [-0 . [] [-0 . []] [-0 . [-0 . []] [-0 . [-0 . []] [-0 . [-0 . [-0 . []] [-0 . [-0 . [-0 . [-0 . [-0 . [-0 . [-0 . [-0 . [| 0 0 0 25 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | - (| 0 | -0. | | 0.5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0.5 0.5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0.5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | -0. | 0 | 0.2 | 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0.25 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0.25 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0] 0] 0] 0] 0] 0] 0] 0] 0] 0] |
| D2Y = | | | | | | | | | | | | | | | | | | | | |
| [[[0.2 [[[[[[[[| | 0 0 0 0 0 .25 0 0 0 0 0 | 0.25 | 0 | 0 0 25 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 -0.5 0 0 0.25 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0.25 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | -0 | 0 0 0 0 0 25 0 0 0 .5 0 0 | 0.25 | 0.25 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 | | |