

## Problem #1: Closed-Form Analysis

A first-order approximation for calculating the tensor elements of a negative uniaxial dielectric metamaterial (i.e. subwavelength planar grating) is

$$[\epsilon_{\text{eff}}] = \begin{bmatrix} \epsilon_o & 0 & 0 \\ 0 & \epsilon_o & 0 \\ 0 & 0 & \epsilon_e \end{bmatrix} \quad (1)$$

$$\epsilon_o \approx f \epsilon_1 + (1-f) \epsilon_2 \quad (2)$$

$$\frac{1}{\epsilon_e} \approx \frac{f}{\epsilon_1} + \frac{1-f}{\epsilon_2} \quad (3)$$

where  $\epsilon_1$  and  $\epsilon_2$  are the two permittivities comprising the structure,  $f$  is the volumetric fill factor of  $\epsilon_1$ ,  $\epsilon_o$  is the effective ordinary permittivity, and  $\epsilon_e$  is the effective extraordinary permittivity.

### Part A

Assuming the tensor in Eq. (1) takes the following form

$$[\epsilon_{\text{eff}}] = \begin{bmatrix} \epsilon a & 0 & 0 \\ 0 & \epsilon a & 0 \\ 0 & 0 & \epsilon/a \end{bmatrix}, \quad (4)$$

derive expressions for  $\epsilon$  and  $a$  in terms of just  $\epsilon_o$ , and  $\epsilon_e$ . Simplify the expressions as much as possible.

### Part B

Using Eqs. (2)-(3) and the results from Part A of this homework, derive expressions for  $\epsilon$  and  $a$  in terms of just  $f$ ,  $\epsilon_1$ , and  $\epsilon_2$ . Simplify the final expressions as much as possible.

### Part C

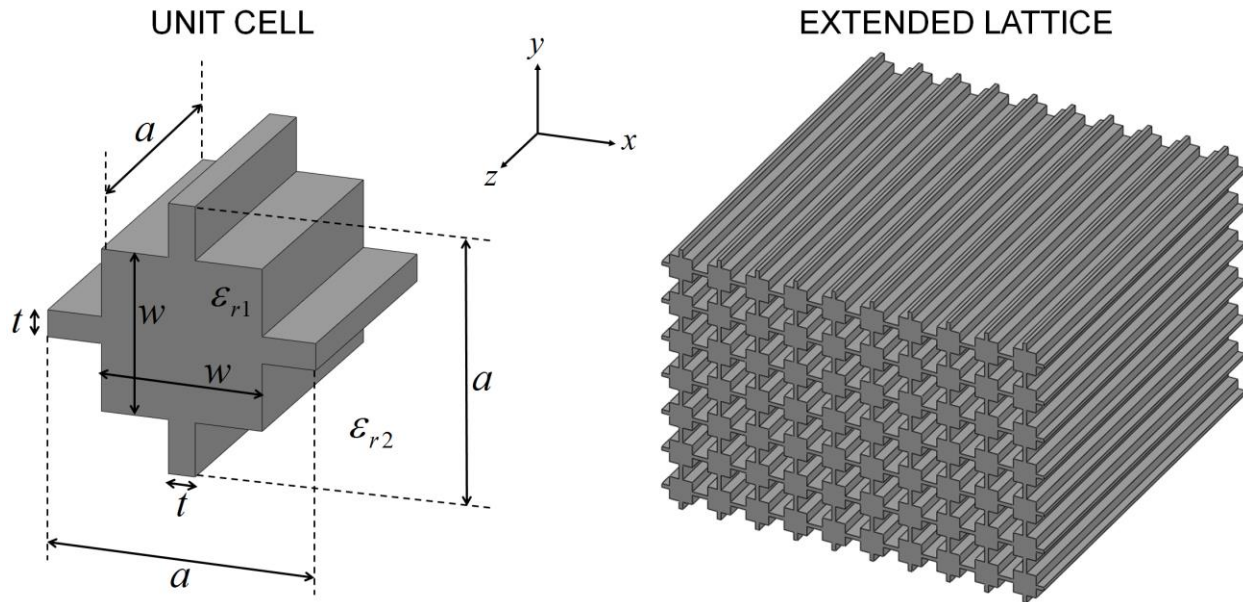
Derive the value of  $f$  that maximizes the value of  $a$ .

### Part D

Derive expressions for both  $a$  and  $\epsilon$  at the optimized value of  $f$  derived in Part C of this homework.

### Problem #2: Numerical Parameter Retrieval

Calculate rigorous values for all three effective dielectric tensor elements for the dielectric anisotropic metamaterial shown below. Assume the lattice is operating in the nonresonant region. You will be graded heavily on the accuracy of your answers!



Parameter	Value
$a$	1.0 cm
$w$	6.0 mm
$t$	1.0 mm
$\epsilon_{r1}$	9.5
$\epsilon_{r2}$	1.0

### Problem #3: Questions

1. Calculate the parameters  $a$  and  $\epsilon$  for the lattice above.
2. What is the cutoff frequency of this metamaterial in GHz? Justify your answer.
3. What type of anisotropy (i.e. isotropic, biaxial, etc.) does this structure provide? Justify your answer.
4. What could be done to increase the parameter  $a$ ? List everything you can think of.