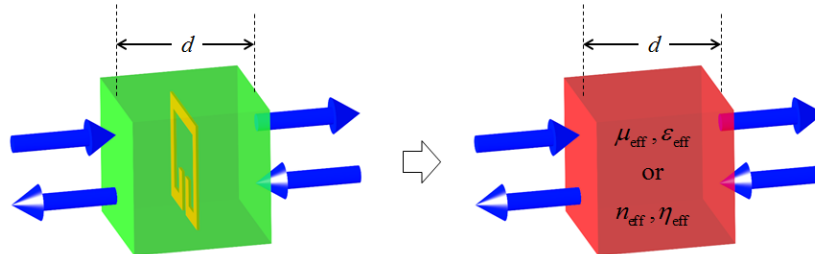


In this homework, it is desired to calculate the effective electromagnetic properties of a metamaterial. To do this, the metamaterial unit cell shown below was manufactured by 3D printing and tested in the lab. The thickness of the unit cell was $d = 5.7$ mm. Using a vector network analyzer, the complex reflection and transmission coefficients, S_{11} and S_{21} respectively, were measured at 10 GHz.



$$S_{11} \text{ is } -9.70 \text{ dB and } 42.74^\circ \text{ phase}$$

$$S_{21} \text{ is } -0.49 \text{ dB and } 132.74^\circ \text{ phase}$$

Problem #1: Calculate Complex S_{11} and S_{21}

The vector network analyzer reports the scattering parameters in terms of magnitude and phase. The magnitude is given as a power quantity in decibels. The phase is given as an angle of phase with units of degrees. This may seem strange, but it is how the measurement equipment typically reports the scattering parameters.

Convert the measured scattering parameter data into the form of complex reflection and transmission coefficients, S_{11} and S_{21} respectively. Perform all calculations by hand and show all work.

$$S_{11} = S'_{11} + jS''_{11} \quad S_{21} = S'_{21} + jS''_{21}$$

Problem #2: Calculate Parameters r and t

Given the complex scattering parameters calculated in Problem #1, calculate the parameters r and t following the NRW method. Perform the calculations by hand and show all work.

Problem #3: Calculate the Effective Impedance and Refractive Index

Given the values of r and t calculated in Problem #2, calculate the effective impedance and effective refractive index following the NRW method. Perform all calculations by hand and show all work.

Problem #4: Calculate the Effective Permeability and Permittivity

Given the effective impedance η_{eff} and effective refractive index n_{eff} calculated in Problem #3, calculate the effective relative permeability $\mu_{r,\text{eff}}$ and effective relative permittivity $\epsilon_{r,\text{eff}}$ following the NRW method. Perform all calculations by hand and show all work.