Surface Waves

Lecture Outline

• Introduction
• Survey of surface waves
• Excitation of surface waves
• Surface plasmon polaritons
• Dyakonov surface waves
Introduction

An infinite half-space is a region in space that is bounded at only one edge. It extends to infinity on all other sides.
Traditional Guided Modes
(1 of 2)

\[\beta = k_0 n_{\text{eff}} = k_0 n \sin \theta\]

Traditional Guided Modes
(2 of 2)
A New Guided Mode – Surface Waves

A surface wave is most analogous to a slab waveguide, but the mode is confined at the interface between two different materials comprising two infinite half spaces. The field decays exponentially away from the interface. It is free to propagate without decay in the plane of the interface.

Why Do Surface Waves Exist?
Recall the Field at an Interface

1. The field always penetrates material 2, but it may not propagate.
2. Above the critical angle, penetration is greatest near the critical angle.
3. Very high spatial frequencies are supported despite the dispersion relation.
4. In material 2, energy always flows along $x$, but not necessarily along $y$. 
Why Do Surface Waves Exist?

Hand Waving Explanation

Wave is cutoff in superstrate.

Propagation is slowed due to increased interaction between the wave and the materials.

Wave is cutoff in substrate

Survey of Surface Waves
Types of Surface Waves

- Zenneck surface waves (ground waves)
- Resonant surface wave
- Surface waves at chiral interfaces
- Surface waves at gyrotropic interfaces
- Nonlinear surface wave
- Surface plasmon polariton (SPP)
- Dyakonov surface wave (DSW)
- Optical Tamm States (OTSs)


Zenneck Surface Wave

Zenneck waves are essentially surface plasmons at RF frequencies.

Fig. 1. Geometry for a vertical magnetic current sheet over a half space of surface impedance Z. The aperture height z₀ can be either finite or infinite.
Norton Surface Wave

Norton waves are vertically polarized (TM) waves supported at the interface between a dielectric and a lossy material. These are also known as ground waves and are why long wavelength signals, such as that from AM radio, travel efficiently across the surface of the earth.

Vertical polarization allows field to extend all the way to the ground.
Diffraction bends the wave along the curved surface.

Resonant Surface Wave

Resonant surface waves exist at the interface between essentially any material and a resonant photonic crystal. The surface waves exist at frequencies lying inside the band gap. Tremendous design freedom is offered by resonant surface waves because the conditions for their existence depend mostly on the geometry of the lattice which can be tailored and adjusted.


Surface Waves at Chiral Interfaces

Chiral materials possess an intrinsic handedness leading to unique electromagnetic properties. Surface waves at the interface of chiral materials are hybrid modes and exhibit split cutoff frequencies. These are attractive features for suppressing surface waves in antennas and for forming directional couplers. They also are excellent absorbers when made of lossy materials.


Surface Waves at Gyrotropic Interfaces

A gyrotropic material is one that is perturbed (Δε) by a quasi-static magnetic field. Surface waves are supported at the interface between a gyrotropic material (either gyroelectric or gyromagnetic) and an isotropic negative phase velocity medium (NPM). These exhibit interesting properties such as nonreciprocal propagation and anomalous dispersion.

Nonlinear Surface Wave

Surface waves can exist at the interface between an ordinary material and a nonlinear material. It has been shown that when the lower refractive index material has a positive Kerr coefficient, the surface wave propagates with perfectly constant shape and intensity and can be excited directly by an external wave.

$D$ is an independent parameter relating the various wave vector components.

$$h_{12} = k_0 \beta_c \sqrt{D},$$
$$h_{22} = k_0 \beta_c \sqrt{1 + D},$$
$$h_4 = k_0 \beta_c \sqrt{\gamma_c^{-2} + D}.$$

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Surface Plasmon-Polariton

Surface plasmon-polaritons (SPPs) are supported at the interface between a material with positive dielectric constant and a material with negative dielectric constant. There exists an analogous surface wave at the interface of a material with negative permeability. Surface plasmons are attracting much attention in the optics community for their very useful propagation characteristics and radical miniaturization, but they suffer from extraordinary losses.
Dyakonov Surface Wave

Dyakonov surface waves (DSWs) are supported at the interface between two materials where at least one is anisotropic. They are not well understood, but exhibit many unique and intriguing properties.


Optical Tamm States

Optical Tamm States (OTSs) can be formed at the interface between two periodic structures. The first has a period close to the wavelength. The second has a period close to the double of the wavelength.

• Superstrate and substrate must have overlapping band gaps, but different periods.
• OTSs exist in any direction along surface.
• Highly sensitive to the order of the layers at the interface.
• Dispersion curve is parabolic.
• May serve as an alternative to DSWs.

Excitation of Surface Waves

Field Visualization for $\theta_c = 45^\circ$

- $\theta_{inc} = 44^\circ$
- $\theta_{inc} = 46^\circ$
- $\theta_{inc} = 67^\circ$
- $\theta_{inc} = 89^\circ$
Conceptual Picture of a Surface Wave

Otto Configuration

Attenuated total reflection setup:

TIR produces a high spatial frequency in Material 1 that matches the propagation constant of the surface wave.

Frequency where surface wave is excited.

Recall the Field Associated with the Diffracted Modes

The wave vector expansion for the first 11 diffracted modes can be visualized as...

\[ k_y \text{ is real. A wave propagates into material 2.} \]

\[ k_y \text{ is imaginary. The field in material 2 is evanescent.} \]

We can use gratings to generate high spatial frequencies.

Grating Coupler Configuration

The grating coupler configuration uses coupled-mode theory to excite a surface wave.

A high-order spatial harmonic (usually the 2nd order) produces a high spatial frequency that matches the propagation constant of the surface wave.
Recall the Field Around a Waveguide

The evanescent field outside of a waveguide has a high spatial frequency and is cutoff by the cladding materials.

Evanescent Coupling Configuration

Cladding
Core
Cladding
Material 1
Material 2
Surface Plasmon Polaritons

Why Do We Care About SPPs?

- Highly subwavelength → radical miniaturization
  - Able to concentrate energy in subwavelength volumes
- Strong dispersive properties → new mechanisms for manipulating waves
  - Able to guide waves along the surface of a metal
- Applications
  - Sensors
  - Data storage
  - Light sources
  - Modulators
  - Microscopy
  - Bio-photonics
  - Subwavelength optics (nanofabrication and imaging)
**Classical Analysis**

We start with Maxwell's equations

\[
\nabla \times \vec{H} = j \omega \varepsilon \vec{E},
\n\nabla \cdot (\varepsilon \vec{E}) = 0
\]

\[
\nabla \times \vec{E} = -j \omega \mu \vec{H},
\n\n\nabla \cdot (\mu \vec{H}) = 0
\]

For 1D geometries, we have

\[
\frac{\partial}{\partial y} = 0
\]

Maxwell's equations split into two independent modes.

\[
\frac{\partial H_z}{\partial z} + j \beta \varepsilon \frac{\partial E_x}{\partial x} = j \omega \mu E_y,
\]

\[
- \frac{\partial E_z}{\partial z} = -j \omega \mu H_y,
\]

\[
\frac{\partial E_x}{\partial x} = -j \omega \mu H_z,
\]

For 1D geometries, we have

\[
\frac{\partial H_x}{\partial x} - j \beta \mu \frac{\partial E_z}{\partial z} = j \omega \varepsilon E_y,
\]

\[
- \frac{\partial E_x}{\partial x} = -j \omega \mu H_z,
\]

\[
\frac{\partial H_z}{\partial z} = j \omega \varepsilon E_y,
\]

**Only the H Mode Exists**

If a wave is to propagate along the surface of a metal, the electric field must be polarized normal to the surface. Otherwise, boundary conditions will require it to be zero. Therefore, the \( E \) mode does not exist.

\[
\frac{\partial H_x}{\partial x} - j \beta \mu \frac{\partial E_z}{\partial z} = j \omega \varepsilon E_y,
\]

\[
- \frac{\partial E_x}{\partial x} = -j \omega \mu H_z,
\]

\[
\frac{\partial H_z}{\partial z} = j \omega \varepsilon E_y,
\]
Assumed Solution

If the wave is a surface wave, it must be confined to the surface. This can only happen if the field decays exponentially away from the interface. This implies the field solution has the following form.

\[
\begin{align*}
\tilde{E}_i(z) &= \left[ E_{1i} e^{-\kappa_1 z} + E_{2i} e^{-\kappa_2 z} \right], \\
\tilde{H}_i(z) &= \left[ H_{1i} e^{-\kappa_1 z} + H_{2i} e^{-\kappa_2 z} \right],
\end{align*}
\]

Substituting this solution into the H mode equations yields

\[
\begin{align*}
\frac{\partial}{\partial z} \left( E_{1i} e^{-\kappa_1 z} \right) - \frac{\partial}{\partial x} \left( E_{1i} e^{-\kappa_1 z} \right) &= -j \omega \mu_0 \mu_r \left( H_{1i} e^{-\kappa_1 z} \right) \\
\frac{\partial}{\partial z} \left( H_{1i} e^{-\kappa_1 z} \right) &= j \omega \epsilon_0 \epsilon_r \left( E_{1i} e^{-\kappa_1 z} \right) \\
\frac{\partial}{\partial x} \left( H_{1i} e^{-\kappa_1 z} \right) &= j \omega \epsilon_0 \epsilon_r \left( E_{1i} e^{-\kappa_1 z} \right)
\end{align*}
\]

Equations in Medium 1

Inside medium 1, our three equations were

\[
\begin{align*}
\frac{\partial}{\partial z} \left( E_{13} e^{-\kappa_1 z} \right) - \frac{\partial}{\partial x} \left( E_{13} e^{-\kappa_1 z} \right) &= -j \omega \mu_0 \mu_r \left( H_{13} e^{-\kappa_1 z} \right) \\
\frac{\partial}{\partial z} \left( H_{13} e^{-\kappa_1 z} \right) &= j \omega \epsilon_0 \epsilon_r \left( E_{13} e^{-\kappa_1 z} \right) \\
\frac{\partial}{\partial x} \left( H_{13} e^{-\kappa_1 z} \right) &= j \omega \epsilon_0 \epsilon_r \left( E_{13} e^{-\kappa_1 z} \right)
\end{align*}
\]

These reduce to

\[
\begin{align*}
\beta E_{13} + \kappa_1 E_{13} &= -j \omega \mu_0 \mu_r H_{13} \\
-j \beta H_{13} &= j \omega \epsilon_0 \epsilon_r E_{13} \\
-\kappa_1 H_{13} &= j \omega \epsilon_0 \epsilon_r E_{13}
\end{align*}
\]
Inside medium 2, our three equations were
\[
\frac{\partial}{\partial z}(E_{z2}e^{j\beta z}) - \frac{\partial}{\partial x}(E_{z2}e^{j\beta x}) = -j\omega\varepsilon_{2}(H_{x2}e^{j\beta x}) \\
-\frac{\partial}{\partial z}(H_{z2}e^{j\beta z}) = j\omega\varepsilon_{2}(E_{z2}e^{j\beta z}) \\
\frac{\partial}{\partial x}(H_{z2}e^{j\beta x}) = j\omega\varepsilon_{2}(E_{z2}e^{j\beta x})
\]
These reduce to
\[
j\beta E_{z2} - \kappa_{z}E_{z2} = -j\omega\varepsilon_{2}\mu_{2}H_{y2} \\
-j\beta H_{y2} = j\omega\varepsilon_{2}\mu_{2}E_{z2} \\
\kappa_{z}H_{y2} = j\omega\varepsilon_{2}\mu_{2}E_{z2}
\]
To more easily match the boundary conditions at \(x=0\), the field component longitudinal to this interface is eliminated from the sets of three equations.

Medium 1
\[
\kappa_{z}E_{z,1} = -j\frac{\omega\varepsilon_{1}\mu_{2}}{\omega\varepsilon_{1}\varepsilon_{2}}(k_{z}^{2}\mu_{2}\varepsilon_{1} - \beta^{2})H_{y,1} \\
\kappa_{z}H_{y,1} = -j\omega\varepsilon_{1}\varepsilon_{2}E_{z,1}
\]
Medium 2
\[
\kappa_{z}E_{z,2} = j\frac{\omega\varepsilon_{1}\mu_{2}}{\omega\varepsilon_{1}\varepsilon_{2}}(k_{z}^{2}\mu_{2}\varepsilon_{2} - \beta^{2})H_{y,2} \\
\kappa_{z}H_{y,2} = j\omega\varepsilon_{1}\varepsilon_{2}E_{z,2}
\]
Dispersion Relation

The dispersion relation is derived by further eliminating $E_z$ and relating the remaining parameters.

Medium 1

$$k_1^2 \mu_1 \varepsilon_1 = \beta^2 - \kappa_1^2$$

Medium 2

$$k_2^2 \mu_2 \varepsilon_2 = \beta^2 - \kappa_2^2$$

This lets us write a general dispersion relation for the $i$th medium as

$$k_0^2 \mu_{r,i} \varepsilon_{r,i} = \beta^2 - \kappa_i^2$$

Boundary Conditions

Electric Field Boundary Conditions

$$E_{r,1} = E_{r,2}$$

$$-j \frac{\omega}{\varepsilon_1 \varepsilon_2} k_3 \left(k_0^2 \mu_1 \varepsilon_{r,1} - \beta^2 \right) H_{r,1} = -j \frac{\omega}{\varepsilon_1 \varepsilon_2} k_3 \left(k_0^2 \mu_2 \varepsilon_{r,2} - \beta^2 \right) H_{r,2}$$

$$\frac{k_1}{\varepsilon_{r,1}} + \frac{k_2}{\varepsilon_{r,2}} = 0$$

Magnetic Field Boundary Conditions

$$H_{r,1} = H_{r,2}$$

$$-j \frac{\omega}{\varepsilon_1 \varepsilon_2} E_{r,1} = -j \frac{\omega}{\varepsilon_1 \varepsilon_2} E_{r,2}$$

$$\frac{E_{r,1}}{k_1} + \frac{E_{r,2}}{k_2} = 0$$

Same equation. Existence Condition.
Existence Condition and Dispersion Relation

The existence condition for a surface plasmon is then

\[ \frac{\varepsilon_{1,1} + \varepsilon_{2,2}}{\kappa_1} = 0 \]

From the dispersion relation in both mediums, we see that

\[ \kappa_1^2 = \beta^2 - k_0^2 \mu_{1,1} \varepsilon_{1,1} \quad \kappa_2^2 = \beta^2 - k_0^2 \mu_{2,2} \varepsilon_{2,2} \]

We can use the dispersion relation to eliminate the \( x \)-terms in the existence condition to obtain the generalized dispersion relation.

\[ \beta^2 = k_0^2 \left( \frac{1}{\varepsilon_{1,1}} - \frac{\mu_{1,1}}{\mu_{2,2}} \right) \left( \frac{\mu_{2,2}}{\varepsilon_{1,1}} - \varepsilon_{2,2} \right) = k_0^2 \left( \frac{\varepsilon_{2,2} \varepsilon_{1,1}}{\varepsilon_{1,1}^2 - \varepsilon_{2,2}^2} \right) (\mu_{2,2} \varepsilon_{1,1} - \mu_{1,1} \varepsilon_{2,2}) \]

For non-magnetic materials, the dispersion relation reduces to

\[ \beta = k_0 \sqrt{\frac{\varepsilon_{2,2} \varepsilon_{1,1}}{\varepsilon_{1,1} + \varepsilon_{2,2}}} \]

Drude Model for Metals

The Drude model for metals was

\[ \varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + j \omega \Gamma} \]

Note, \( N \) is now interpreted as electron density \( N_e \).

\[ \omega_p^2 = \frac{N q^2}{\varepsilon_0 m_e} \]

\( m_e \) is the effective mass of the electron.
Plasmons Require Metals

The existence condition is
\[ \frac{\varepsilon_{r,1}}{\kappa_1} + \frac{\varepsilon_{r,2}}{\kappa_2} = 0 \]

This can be solved for \( \varepsilon_{r,2} \) as follows.
\[ \varepsilon_{r,2} = -\varepsilon_{r,1} \frac{\kappa_2}{\kappa_1} \]

\( \kappa_1 \) and \( \kappa_2 \) are both positive quantities. This shows that \( \varepsilon_{r,1} \) and \( \varepsilon_{r,2} \) must have opposite sign to support a surface wave.

How do we get a negative \( \varepsilon \)? We use metals!!

Surface Plasma Frequency, \( \omega_{sp} \)

For very small damping factor \( \Gamma \), the Drude model reduces to
\[ \varepsilon_{r,2} = 1 - \frac{\omega_p^2}{\omega^2} \]

We can derive an expression for the “surface plasma frequency” by substituting this equation into the dispersion relation, letting \( \omega = \omega_{sp} \), and taking the limit as \( \beta \to \infty \).

\[ \beta = \frac{\omega_p}{c_0} \sqrt{\varepsilon_{r,1} \varepsilon_{r,2}} \]

\[ \omega_{sp} = \frac{\omega_p}{\sqrt{1 + \varepsilon_{r,1}}} \]

Note: we can have SPPs at all frequencies below \( \omega_{sp} \).
Dispersion Relation for a SPP

\[
\omega_p = \frac{\omega}{\sqrt{1 + \varepsilon_2}}
\]

Surface plasma frequency

\[
\beta = \frac{\omega}{c_0}
\]

Propagation constant \( \beta \)

SPPs do not exist at frequencies above \( \omega_p \)

SPPs exist

\[
\beta = k_0 \sqrt{\frac{\varepsilon_{r,1}\varepsilon_{r,2}}{\varepsilon_{r,2} + \varepsilon_{r,1}}}
\]

Normalized Frequency \( \omega / \omega_p \)

SPP behaves very photon like

Excitation of SPPs: Otto Configuration

Attenuated total reflection setup

TIR produces a high spatial frequency in Material 1 that matches the propagation constant of the surface wave.

Material 1

Material 2

http://www.bionavis.com/technology/spi/

Excitation of SPPs: Kretschmann Configuration

Attenuated total reflection setup


Excitation of SPPs: Grating Coupler Configuration

The grating coupler configuration uses coupled-mode theory to excite a surface wave.
Plasmonic Waveguides and Circuits


Band Gap Structures for SPPs

Dyakonov Surface Waves

What is a DSW?

- A DSW is a surface wave confined at the interface between two materials where at least one is anisotropic.
- Anisotropy can be produced by nonresonant metamaterials.
- Nonresonant nature suggest a very broadband phenomenon.
- Note that the peak of the mode is shifted into the anisotropic substrate.
Benchmark Example

For this surface wave analysis, I chose...

\[ \begin{bmatrix} \varepsilon_{z1} \\ \varepsilon_{z2} \end{bmatrix} = \begin{bmatrix} 3.9850 \\ 3.9850 \end{bmatrix} \]

\[ \begin{bmatrix} 3.9850 \\ 0 \end{bmatrix} \]

\[ \begin{bmatrix} 0 \\ 7.6210 \end{bmatrix} \]

Rotated \( \theta = 54^\circ \) about x-axis

\[ \lambda_0 = 1 \]

\[ \varepsilon_x = 3.985 \]

\[ \varepsilon_z = 7.621 \]

\[ \varepsilon_{z1} = \frac{\varepsilon_x + \varepsilon_z}{2} = 5.803 \]

\[ \theta = 54^\circ \]

What Does a DSW Look Like?

Lecture 21
Existence Conditions

The most common configuration for a DSW is a uniaxial substrate and an isotropic superstrate.

The uniaxial substrate must have positive birefringence.

\[ n_O < n_E \]

Superstrate must have a refractive index between \( n_E \) and \( n_O \).

\[ n_O < n_s < n_E \]

The surface wave can only propagate within a narrow range of angles relative to the optical axis.

Angular Existence Domain

The minimum and maximum angles are

\[
\sin^2 \theta_{\text{min}} = \frac{\xi}{2} \left[ \left( 1 - \rho \xi \right) + \sqrt{\left( 1 - \rho \xi \right)^2 + 4 \rho} \right]
\]

\[
\sin^2 \theta_{\text{max}} = \frac{\xi (1 + \rho)^3}{(1 + \rho)^2(1 + \rho \xi) - \rho^2 (1 - \xi)^2}
\]

The central angle is

\[
\theta_0 \approx \sin^{-1} \left[ \frac{\xi (1 + \rho)}{\sqrt{\xi \rho + 1}} \right]
\]
Metamaterial Substrate

Much stronger anisotropy can be realized using metamaterials. This can widen the existence domain and provide a mechanism for sculpting the anisotropy to form more advanced devices.

Conceptual Metamaterial Structures Supporting DSWs
Finite Thickness Superstrate and Substrate

A finite thickness superstrate narrows the existence domain.

Some preliminary research suggests that a finite thickness substrate widens the existence domain, but it is not clear where the cutoff is between a DSW and an ordinary mode guided in the slab.

DSW Dispersion

DSWs exhibit very low dispersion.

(c) dispersion comparison between DSW and a standard slab mode
Excitation of a DSW

DSWs can be excited the same ways as surface plasmons. At radio and microwave frequencies, grating couplers might be preferred due to their compact size and options for integration.