**Problem #1: Waveguide Gap**

A integrated optical waveguide is fabricated onto a fused silica substrate. First a thick layer of silicon nitride is deposited onto the substrate and then lithography is used to etch the structure shown below. The width of the waveguide is 750 nm, the height is 750 nm, and it is operated at the standard telecommunications wavelength of 1.5 μm. Further, a gap of length \( g \) is etched into the waveguide where some of the light scatters out of the waveguide.

Write a finite-difference time-domain (FDTD) program in MATLAB to simulate this waveguide circuit. Use your FDTD program to design the gap length so that exactly half of the optical power is transmitted through the gap and into the second portion of the waveguide. Provide the following:

1. Identify and discuss all simplifications or approximations you make in order to obtain your solution.
2. Report the grid resolution where your simulation converges.
3. Gap length \( g \) where power in the fundamental mode of the output waveguide is 50%.
4. The steady-state field at \( \lambda_0 = 1.5 \) μm across the entire grid. Superimpose the waveguide structure and PML.

**Hint**: Reduce this to a 2D problem using the effective index method. The effective refractive of the vertical cross section through the core is 1.8 and the effective refractive index is 1.4 everywhere else.