

Problem #1 – Derive Update Equations

Starting with Maxwell's equations in the following form

$$\nabla \cdot \vec{D}(t) = \rho_v(t), \quad (1)$$

$$\nabla \cdot \vec{B}(t) = 0, \quad (2)$$

$$\nabla \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t}, \quad (3)$$

$$\nabla \times \vec{H}(t) = \vec{J} + \frac{\partial \vec{D}(t)}{\partial t}, \quad (4)$$

and the constitutive relations

$$\vec{D}(t) = [\varepsilon(t)] * \vec{E}(t), \quad (5)$$

$$\vec{B}(t) = [\mu(t)] * \vec{H}(t), \quad (6)$$

derive the update equations for a one-dimensional finite-difference time-domain algorithm as outlined in Lecture 5. Organize the derivation, explain each step, and show as much detail as possible. Summarize the final update equations at the end and write them in the order they should be implemented (see Block diagram at the end of Lecture 5). **DO NOT COPY/PASTE ANYTHING FROM THE LECTURE NOTES.**

Suggestions:

1. Over the course of several homework assignments, you will evolve this formulation into a comprehensive formulation of the entire FDTD algorithm including boundary conditions, sources, stability, and more. Write this formulation in a computer document that you can continually revise. If you hand write this assignment, you will have to repeat a lot of work!!
2. Write this document for yourself. Include whatever extra notes or diagrams you wish that will help you understand the formulation of FDTD.

Problem #2 – Implement Update Equations in MATLAB

Write a clean and well commented MATLAB program that implements a one-dimensional FDTD algorithm for the E_y/H_x mode. Use the following parameters:

$$\begin{array}{lll} N_z = 180 & \Delta z = 1.4286 \times 10^{-8} \text{ m} & \epsilon_r(z) = 1.0 \\ \text{STEPS} = 1000 & \Delta t = 4.7652 \times 10^{-17} \text{ sec} & \mu_r(z) = 1.0 \end{array}$$

Here, N_z is the total number of points in the 1D grid. Δz is the grid resolution parameter. Δt is the duration of each time step. STEPS is the total number of iterations. $\epsilon_r(z)$ and $\mu_r(z)$ are the relative permittivity and relative permeability functions along the z -axis. Later in the course we will learn how to calculate these parameters given the simulation we wish to perform. **Do not assume these are correct for all situations. They are not!**

Your program should perform the following:

- Define the grid parameters and material properties at the start of the program (dashboard).
- Compute the update coefficients before the main FDTD loop starts.
- Initialize the fields to zero before the main FDTD loop starts.
- Iterate over STEPS number of iterations and perform the following functions at each iteration:
 - Update H from E
 - Update E from H
- Assume field values outside of the grid (if needed) are zero.
- Plot the E and H fields after the main loop is over and make the figure look professional.
- Follow the block diagram at the end of Lecture 5 exactly!