Problem #1: Maxwell's Equations with a PML

Starting with Maxwell's equations

$$\nabla \bullet \vec{D}(t) = \rho_v(t), \tag{1}$$

$$\nabla \bullet \vec{B}(t) = 0, \tag{2}$$

$$\nabla \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t}, \tag{3}$$

$$\nabla \times \vec{H}(t) = \vec{J} + \frac{\partial \vec{D}(t)}{\partial t}, \tag{4}$$

and the constitutive relations

$$\vec{D}(t) = \left[\varepsilon(t)\right] * \vec{E}(t), \tag{5}$$

$$\vec{B}(t) = \left[\mu(t)\right] * \vec{H}(t), \tag{6}$$

derive the final form of Maxwell's equations for the H-D-E formulation that includes a uniaxial perfectly matched layer (UPML) absorbing boundary condition. The final equations are provided in Lecture 13. Include a detailed derivation and explanation of the UPML in the frequency domain. You do not have to derive the Fresnel equations.

DO NOT COPY/PASTE ANYTHING FROM THE LECTURE NOTES. Perform and explain each detailed step of the derivation yourself.

Problem #2: 3D-FDTD Update Equations with a PML

Starting with the governing equations derived in Problem #1, perform detailed derivations of the update equations for H_z, D_z, and E_z. The remaining field components (H_x, H_y, E_x, E_y, D_x, and D_y) may be written by analogy. Please show every step of the derivations.

Problem #3: Outline and/or Block Diagram

Within the space of one page, summarize the basic 2D FDTD algorithm (not the formulation) in a block diagram or in a step-by-step outline with equations. This should include initialize steps, update coefficients, integration terms, curl terms, update equations, and anything covered in Lecture 14.

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