Lecture Outline

- Review of Lecture 4
- Yee grid scheme
- Finite-difference approximation of Maxwell’s curl equations
- Governing equations for one-dimensional FDTD
- Derivation of basic update equations
- Implementation of basic update equations for $E_y/H_x$ mode
Review of Lecture #4

Maxwell’s Equations

**Gauss’ Law**

\[ \nabla \cdot \mathbf{D} = \rho \]

Electric fields arise from positive charges and converge on negative charges.

**Gauss’ Law for Magnetic Fields**

\[ \nabla \cdot \mathbf{B} = 0 \]

If there are no charges, electric fields must form loops.

**Faraday’s Law**

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

Circulating electric fields induce time-varying magnetic fields.

**Ampere’s Circuit Law**

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]

Circulating magnetic fields induce currents and/or time-varying electric fields. Currents and/or time-varying electric fields induce circulating magnetic fields.
Summary of Parameter Relations

Permittivity

\[ \varepsilon = \varepsilon_0 \varepsilon_r \]
\[ \varepsilon_0 = 8.854187817 \times 10^{-12} \text{ F/m} \]

Permeability

\[ \mu = \mu_0 \mu_r \]
\[ \mu_0 = 1.256637061 \times 10^{-6} \text{ H/m} \]

Refractive Index

\[ n = \sqrt{\mu \varepsilon_r} \]
\[ n = \sqrt{\varepsilon_r} \text{ no magnetic response} \]

Impedance

\[ \eta = \eta_0 \sqrt{\mu_r / \varepsilon_r} \]
\[ \eta_0 = \sqrt{\mu_0 / \varepsilon_0} = 376.73031346177 \Omega \]

Wave Velocity

\[ v = \frac{c_0}{n} \]
\[ c_0 = 299792458 \text{ m/s} \]

Flow of Maxwell’s Equations

\[ \nabla \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t} \]
A circulating \( \vec{E} \) field induces a change in the \( \vec{B} \) field at the center of circulation.

\[ \vec{B}(t) = \left[ \mu(t) \right] \ast \vec{H}(t) \]
A \( \vec{B} \) field induces an \( \vec{H} \) field in proportion to the permeability.

\[ \vec{D}(t) = \left[ \varepsilon(t) \right] \ast \vec{E}(t) \]
A \( \vec{D} \) field induces an \( \vec{E} \) field in proportion to the permittivity.

\[ \nabla \times \vec{H}(t) = \frac{\partial \vec{D}(t)}{\partial t} \]
A circulating \( \vec{H} \) field induces a change in the \( \vec{D} \) field at the center of circulation.
Finite-Difference Approximation of Maxwell’s Equations

We approximated the time derivatives in Maxwell’s equations with finite-differences.

\[
\nabla \times \vec{E}(t) = -\mu \frac{\partial \vec{H}(t)}{\partial t} \quad \Rightarrow \quad \nabla \times \vec{E} = -\mu \frac{\vec{H}_{t+\Delta t/2} - \vec{H}_{t-\Delta t/2}}{\Delta t}
\]

\[
\nabla \times \vec{H}(t) = \varepsilon \frac{\partial \vec{E}(t)}{\partial t} \quad \Rightarrow \quad \nabla \times \vec{H} = \varepsilon \frac{\vec{E}_{t+\Delta t/2} - \vec{E}_{t-\Delta t/2}}{\Delta t}
\]

The FDTD Update Equation

Field at the next time step.
Field at the previous time step.
Curl of the “other” field at an intermediate time step.

To speed simulation, we calculate these before iteration.
The FDTD Algorithm...for now 😊

Initialize Fields to Zero
\[ \vec{E} = \vec{H} = 0 \]

Loop over time

Done? 

no

Update \( \vec{H} \) from \( \vec{E} \)
\[ \vec{H} \big|_{t+\Delta t/2} = \vec{H} \big|_{t-\Delta t/2} - \frac{\Delta t}{\mu} (\nabla \times \vec{E}) \]

yes

Finished!

Update \( \vec{E} \) from \( \vec{H} \)
\[ \vec{E} \big|_{t+\Delta t} = \vec{E} \big|_{t} + \frac{\Delta t}{\varepsilon} (\nabla \times \vec{H}) \big|_{t+\Delta t/2} \]

Yee Grid Scheme
Representing Functions on a Grid

- Example physical (continuous) 2D function
- A grid is constructed by dividing space into discrete cells
- Function is known only at discrete points
- Representation of what is actually stored in memory

Grid Unit Cell

A function value is assigned to a specific point within the grid unit cell.
A three-dimensional grid looks like this:

\[ N_x = 10, N_y = 10, N_z = 15 \]

Within the unit cell, we need to place the field components \( E_x, E_y, E_z, H_x, H_y, \) and \( H_z. \)

A straightforward approach would be to locate all of the field components within in a grid cell at the origin of the cell.
Instead, we are going to stagger the position of each field component within the grid cells.

Reasons to Use the Yee Grid Scheme

1. Divergence-free
   \[ \nabla \cdot (\varepsilon \vec{E}) = 0 \]
   \[ \nabla \cdot (\mu \vec{H}) = 0 \]

2. Physical boundary conditions are naturally satisfied

3. Elegant arrangement to approximate Maxwell’s curl equations

Yee Cell for 1D, 2D, and 3D Grids

3D Yee Grid

2D Yee Grids

1D Yee Grid
Consequences of the Yee Grid

- Field components are in physically different locations
- Field components may reside in different materials even if they are in the same unit cell
- Field components will be out of phase
- Recall the field components are also staggered in time

Visualizing Extended Yee Grids

4×4×4 Grid

4×4 Grid (E Mode)
Finite-Difference Approximation to Maxwell’s Equations

Normalize the Magnetic Field

We satisfied the divergence equations by adopting the Yee grid scheme. We now only have to deal with the curl equations.

\[ \nabla \times \vec{E} = -[\mu] \frac{\partial \vec{H}}{\partial t} \quad \nabla \times \vec{H} = [\varepsilon] \frac{\partial \vec{E}}{\partial t} \]

The \( \vec{E} \) and \( \vec{H} \) fields are related through the impedance of the material they are in, so they are roughly three orders of magnitude different.

\[ |\vec{E}| \approx \eta |\vec{H}| \quad \eta \approx 300 \, \Omega \]

This may cause rounding errors in your simulation and it is always good practice to normalize your parameters so they are all the same order of magnitude. Here we choose to normalize the magnetic field.

\[ |\vec{E}| \approx |\vec{H}| \quad \rightarrow \quad \vec{H} = \eta \vec{\tilde{H}} \quad \vec{\tilde{H}} = \eta_0 \vec{\tilde{H}} \]

We do not know the actual impedance ahead of time, so we just use the free space impedance.
Using the normalized magnetic field, the curl equations become

\[
\nabla \times \vec{E} = -\left[ \frac{\mu_r}{c_0} \right] \frac{\partial \vec{H}}{\partial t}
\]

\[
\nabla \times \vec{H} = \left[ \frac{\epsilon_r}{c_0} \right] \frac{\partial \vec{E}}{\partial t}
\]

**Proof**

\[
\vec{H} = \eta_0 \vec{H}_0 \quad \rightarrow \quad \vec{H} = \frac{\vec{H}}{\eta_0}
\]

**Note:**

\[
\begin{align*}
\left[ \mu \right] &= \mu_0 \left[ \mu_r \right] \\
\left[ \epsilon \right] &= \epsilon_0 \left[ \epsilon_r \right] \\
\eta_0 &= \sqrt{\frac{\mu_0}{\epsilon_0}} \\
c_0 &= \frac{1}{\sqrt{\mu_0 \epsilon_0}}
\end{align*}
\]

---

**Expand the Curl Equations**

\[
\nabla \times \vec{E} = \left[ \frac{\mu_r}{c_0} \right] \frac{\partial \vec{H}}{\partial t}
\]

\[
\begin{align*}
\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial z} &= \frac{1}{c_0} \left( \frac{\partial H_z}{\partial t} + \mu_0 \frac{\partial H_y}{\partial t} + \mu_0 \frac{\partial H_z}{\partial t} \right) \\
\frac{\partial E_z}{\partial z} - \frac{\partial E_z}{\partial z} &= \frac{1}{c_0} \left( \frac{\partial H_y}{\partial t} + \mu_0 \frac{\partial H_y}{\partial t} + \mu_0 \frac{\partial H_z}{\partial t} \right) \\
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= \frac{1}{c_0} \left( \frac{\partial H_z}{\partial t} + \mu_0 \frac{\partial H_y}{\partial t} + \mu_0 \frac{\partial H_z}{\partial t} \right)
\end{align*}
\]

\[
\nabla \times \vec{H} = \left[ \frac{\epsilon_r}{c_0} \right] \frac{\partial \vec{E}}{\partial t}
\]

\[
\begin{align*}
\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial y} &= \frac{1}{c_0} \left( \epsilon_{xx} \frac{\partial E_x}{\partial t} + \epsilon_{xy} \frac{\partial E_y}{\partial t} + \epsilon_{yx} \frac{\partial E_y}{\partial t} \right) \\
\frac{\partial H_y}{\partial z} - \frac{\partial H_y}{\partial z} &= \frac{1}{c_0} \left( \epsilon_{yy} \frac{\partial E_y}{\partial t} + \epsilon_{yz} \frac{\partial E_z}{\partial t} + \epsilon_{zy} \frac{\partial E_z}{\partial t} \right) \\
\frac{\partial H_z}{\partial z} - \frac{\partial H_z}{\partial z} &= \frac{1}{c_0} \left( \epsilon_{zz} \frac{\partial E_z}{\partial t} + \epsilon_{xz} \frac{\partial E_x}{\partial t} + \epsilon_{zz} \frac{\partial E_z}{\partial t} \right)
\end{align*}
\]
Assume Only Diagonal Tensors

\[ \nabla \times \vec{E} = -\left[ \mu_r \right] \frac{\partial \vec{B}}{\partial t} \]
\[ \nabla \times \vec{H} = \left[ \varepsilon_r \right] \frac{\partial \vec{D}}{\partial t} \]

Final Analytical Equations

These are the final form of Maxwell’s equations from which we will formulate the FDTD method.

\[ \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\mu_{xx}}{c_0} \frac{\partial H_x}{\partial t} \]
\[ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\mu_{yy}}{c_0} \frac{\partial H_y}{\partial t} \]
\[ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\mu_{zz}}{c_0} \frac{\partial H_z}{\partial t} \]

Next, we will approximate these equations with finite-differences in the Yee grid.
Finite-Difference Equation for $H_x$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\mu_{xx}}{c_0} \frac{\partial \tilde{H}_x}{\partial t}$$

$$\frac{E_{z}^{i,j,k+1} - E_{z}^{i,j,k}}{\Delta y} - \frac{E_{y}^{i,j,k+1} - E_{y}^{i,j,k}}{\Delta z} = -\frac{\mu_{xx}}{c_0} \frac{\tilde{H}_x^{i,j,k+1}}{\Delta t} - \frac{\tilde{H}_x^{i,j,k}}{\Delta t}$$

Finite-Difference Equation for $H_y$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\mu_{yy}}{c_0} \frac{\partial \tilde{H}_y}{\partial t}$$

$$\frac{E_{x}^{i,j,k+1} - E_{x}^{i,j,k}}{\Delta z} - \frac{E_{z}^{i+1,j,k} - E_{z}^{i,j,k}}{\Delta x} = -\frac{\mu_{yy}}{c_0} \frac{\tilde{H}_y^{i,j,k+1}}{\Delta t} - \frac{\tilde{H}_y^{i,j,k}}{\Delta t}$$
Finite-Difference Equation for $H_z$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\mu_{zz}}{c_0} \frac{\partial \tilde{H}_z}{\partial t}$$

$$\frac{E_y^{i+1,j,k} - E_y^{i,j,k}}{\Delta x} - \frac{E_x^{i,j,k+1} - E_x^{i,j,k}}{\Delta y} = -\frac{\mu_{zz}}{c_0} \frac{\tilde{H}_z^{i,j,k} - \tilde{H}_z^{i,j,k-1}}{\Delta t}$$

Finite-Difference Equation for $E_x$

$$\frac{\partial \tilde{H}_z}{\partial y} - \frac{\partial \tilde{H}_y}{\partial z} = \frac{\varepsilon_{xx}}{c_0} \frac{\partial E_x}{\partial t}$$

$$\frac{\tilde{H}_z^{i,j,k} - \tilde{H}_z^{i,j,k-1}}{\Delta y} - \frac{\tilde{H}_y^{i,j,k} - \tilde{H}_y^{i,j,k-1}}{\Delta z} = -\frac{\varepsilon_{xx}}{c_0} \frac{E_x^{i,j,k} - E_x^{i,j,k-1}}{\Delta t}$$
Finite-Difference Equation for $E_y$

\[ \frac{\partial \tilde{H}_x}{\partial z} - \frac{\partial \tilde{H}_z}{\partial x} = \frac{\varepsilon_{yy}}{c_0} \frac{\partial E_y}{\partial t} \]

\[ \tilde{H}_{x,ij,k}^{i,j,k} = \tilde{H}_{x,ij,k-1}^{i,j,k} \frac{\Delta z}{\Delta z} - \tilde{H}_{z,ij,k}^{i,j,k} \frac{\Delta z}{\Delta z} = \frac{\varepsilon_{yy}}{c_0} \frac{E_y^{i,j,k}}{\Delta t} \]

Lecture 5

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Finite-Difference Equation for $E_z$

\[ \frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} = \frac{\varepsilon_{zz}}{c_0} \frac{\partial E_z}{\partial t} \]

\[ \tilde{H}_{y,ij,k}^{i,j,k} = \tilde{H}_{y,ij,k-1}^{i,j,k} \frac{\Delta x}{\Delta x} - \tilde{H}_{z,ij,k}^{i,j,k} \frac{\Delta x}{\Delta x} = \frac{\varepsilon_{zz}}{c_0} \frac{E_z^{i,j,k}}{\Delta t} \]

Lecture 5

Slide 32
Summary of Finite-Difference Equations

Each equation is enforced separately for each cell in the grid. This is repeated for each time step until the simulation is finished. These equations get repeated a lot!!
Reduction to One Dimension

We saw in Lecture 3 that some problems composed of dielectric slabs can be described in just one dimension. In this case, the materials and the fields are uniform in two directions. Derivatives in these uniform directions will be zero. We will define the uniform directions to be the $x$ and $y$ axes.

\[
\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0
\]

$x$ and $y$ Derivatives are Zero
Maxwell’s Equations Decouple Into Two Independent Modes

We see that the longitudinal field components $E_z$ and $H_z$ are always zero. We also see that Maxwell’s equations have decoupled into two sets of two equations.

\[
\begin{align*}
\frac{\partial E_x}{\partial z} &= \mu_0 \frac{\partial H_y}{\partial t} \\
\frac{\partial E_y}{\partial z} &= \mu_0 \frac{\partial H_x}{\partial t} \\
0 &= \varepsilon_0 \frac{\partial E_z}{\partial t} \\
\frac{\partial H_y}{\partial z} &= \varepsilon_0 \frac{\partial E_x}{\partial t}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial E_x}{\partial z} - E^{j,k}_{x} &= \mu_0 \frac{\partial H_y}{\partial t} - H^{j,k}_{y} \\
\frac{\partial E_y}{\partial z} - E^{j,k}_{y} &= \mu_0 \frac{\partial H_x}{\partial t} - H^{j,k}_{x} \\
0 &= \varepsilon_0 \frac{\partial E_z}{\partial t} \\
\frac{\partial H_y}{\partial z} - H^{j,k}_{y} &= \varepsilon_0 \frac{\partial E_x}{\partial t} - E^{j,k}_{x}
\end{align*}
\]

Two Remaining Modes are the Same

We see that the longitudinal field components $E_y$ and $H_x$ are always zero. We also see that Maxwell’s equations have decoupled into two sets of two equations.

\[
\begin{align*}
\frac{\partial E_x}{\partial z} &= \mu_0 \frac{\partial H_y}{\partial t} \\
\frac{\partial E_y}{\partial z} &= \mu_0 \frac{\partial H_x}{\partial t} \\
0 &= \varepsilon_0 \frac{\partial E_z}{\partial t} \\
\frac{\partial H_x}{\partial z} &= \varepsilon_0 \frac{\partial E_y}{\partial t}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial E_x}{\partial z} - E^{j,k}_{x} &= \mu_0 \frac{\partial H_y}{\partial t} - H^{j,k}_{y} \\
\frac{\partial E_y}{\partial z} - E^{j,k}_{y} &= \mu_0 \frac{\partial H_x}{\partial t} - H^{j,k}_{x} \\
0 &= \varepsilon_0 \frac{\partial E_z}{\partial t} \\
\frac{\partial H_x}{\partial z} - H^{j,k}_{x} &= \varepsilon_0 \frac{\partial E_y}{\partial t} - E^{j,k}_{y}
\end{align*}
\]

While these modes are physical and would propagate independently, they are numerically the same and will exhibit the same electromagnetic behavior in isotropic media. Therefore, it is only necessary to solve one. We will proceed with the $E_y/H_x$ mode.
No Longer Need $i$ and $j$ Array Indices

E$_y$/H$_y$ Mode

$$\frac{\tilde{H}^k_{y, z+\frac{1}{2}} - \tilde{H}^{k-1}_{y, z+\frac{1}{2}}}{\Delta z} = \frac{\varepsilon^k_{yy} E^k_{y, z+\frac{1}{2}} - E^k_{y, z}}{c_0 \Delta t}$$

$$\frac{E^{k+1}_{y, z} - E^k_{y, z}}{\Delta z} = \frac{\mu^k_{yy} \tilde{H}^k_{y, z+\frac{1}{2}} - \tilde{H}^k_{y, z}}{c_0 \Delta t}$$

E$_y$/H$_x$ Mode

$$\frac{E^{k+1}_{y, z} - E^k_{y, z}}{\Delta z} = \frac{\mu^k_{xx} \tilde{H}^k_{y, z+\frac{1}{2}} - \tilde{H}^k_{y, z}}{c_0 \Delta t}$$

$$\frac{\tilde{H}^k_{y, z+\frac{1}{2}} - \tilde{H}^{k-1}_{y, z+\frac{1}{2}}}{\Delta z} = \frac{\varepsilon^k_{yx} E^k_{y, z+\frac{1}{2}} - E^k_{y, z}}{c_0 \Delta t}$$

Derivation of the Basic Update Equations
Update Equation for $E_x$

Start with the finite-difference equation which has $E_x$ in the time-derivative:

$$
\frac{\hat{H}_x^{i+1} - \hat{H}_x^i}{\Delta t} = \frac{\varepsilon_{xx}^i}{c_0} \frac{E_x^{i+1} - E_x^i}{\Delta t}
$$

Solve this for $E_x$ at the future time value.

$$
E_x^{i+1} = E_x^i + \left( \frac{c_0 \Delta t}{\varepsilon_{xx}^i} \right) \left( \frac{\hat{H}_x^{i+1} - \hat{H}_x^i}{\Delta t} \right)
$$

Update Equation for $E_y$

Start with the finite-difference equation which has $E_y$ in the time-derivative:

$$
\frac{\hat{H}_y^{i+1} - \hat{H}_y^i}{\Delta t} = \frac{\varepsilon_{yy}^i}{c_0} \frac{E_y^{i+1} - E_y^i}{\Delta t}
$$

Solve this for $E_y$ at the future time value.

$$
E_y^{i+1} = E_y^i + \left( \frac{c_0 \Delta t}{\varepsilon_{yy}^i} \right) \left( \frac{\hat{H}_y^{i+1} - \hat{H}_y^i}{\Delta t} \right)
$$
Update Equation for $H_x$

Start with the finite-difference equation which has $H_x$ in the time-derivative:

$$\frac{E_x^{i+1} - E_x^i}{\Delta z} = \frac{\mu_x}{c_0} \frac{\tilde{H}_x^{i+1} - \tilde{H}_x^i}{\Delta t}$$

Solve this for $H_x$ at the future time value.

$$\frac{\mu_x}{c_0} \frac{\tilde{H}_x^{i+1} - \tilde{H}_x^i}{\Delta t} = \frac{E_x^{i+1} - E_x^i}{\Delta z}$$

$$\tilde{H}_x^{i+1} - \tilde{H}_x^i = \frac{c_0 \Delta t}{\mu_x} \left( \frac{E_x^{i+1} - E_x^i}{\Delta z} \right)$$

$$\tilde{H}_x^{i+1} = \tilde{H}_x^i + \frac{c_0 \Delta t}{\mu_x} \left( \frac{E_x^{i+1} - E_x^i}{\Delta z} \right)$$

Update Equation for $H_y$

Start with the finite-difference equation which has $H_y$ in the time-derivative:

$$\frac{E_y^{i+1} - E_y^i}{\Delta z} = \frac{\mu_y}{c_0} \frac{\tilde{H}_y^{i+1} - \tilde{H}_y^i}{\Delta t}$$

Solve this for $H_y$ at the future time value.

$$\frac{\mu_y}{c_0} \frac{\tilde{H}_y^{i+1} - \tilde{H}_y^i}{\Delta t} = \frac{E_y^{i+1} - E_y^i}{\Delta z}$$

$$\tilde{H}_y^{i+1} - \tilde{H}_y^i = \frac{c_0 \Delta t}{\mu_y} \left( \frac{E_y^{i+1} - E_y^i}{\Delta z} \right)$$

$$\tilde{H}_y^{i+1} = \tilde{H}_y^i + \frac{c_0 \Delta t}{\mu_y} \left( \frac{E_y^{i+1} - E_y^i}{\Delta z} \right)$$
Implementation of the Basic Update Equations for the $E_y/H_x$ Mode

Efficient Implementation of the Update Equations

The update coefficients do not change their value during the simulation. They should be computed only once before the main FDTD loop and not at each iteration inside the loop.

The finite-difference equations in terms of the update coefficients are:

$$
\begin{align*}
\tilde{H}_x^{k} \big|_{l+\Delta l} &= \tilde{H}_x^{k} \big|_{l-\Delta l} + m_{Hx}^{k} \left( \frac{E_y^{k+1} \big|_{l} - E_y^{k} \big|_{l}}{\Delta z} \right) \\
E_y^{k} \big|_{l+\Delta l} &= E_y^{k} \big|_{l} + m_{Ey}^{k} \left( \frac{\tilde{H}_x^{k} \big|_{l+\Delta l} - \tilde{H}_x^{k} \big|_{l-\Delta l}}{\Delta z} \right)
\end{align*}
$$

$H_y$, $E_y$, $E_y$, $\mu_{xx}$, $m_{Hx}$, and $m_{Ey}$ are all stored in 1D arrays of length $N_z$.

$c_0$, $\Delta t$, and $\Delta z$ are single scalar numbers, not arrays.

$$
\begin{align*}
m_{Ey}^{k} &= \frac{c_0 \Delta t}{\varepsilon_y} \\
m_{Hx}^{k} &= \frac{c_0 \Delta t}{\mu_{xx}}
\end{align*}
$$
### The Basic 1D-FDTD Algorithm

**Initialize Fields to Zero**

\[ \widetilde{E} = \widetilde{H} = 0 \]

Calculate the update coefficients before the main loop to save computations.

\[ \widetilde{H}^k = \widetilde{E}^k = 0 \quad \text{for all } k \]

**Update H from E**

\[ \bar{H}^k_{z+\Delta z} = \bar{H}^k_{z-\Delta z} + \frac{\Delta t}{\mu} \left( \nabla \times \bar{E}^k_{z-\Delta z} \right) \]

**Update E from H**

\[ \bar{E}^k_{z+\Delta z} = \bar{E}^k_{z-\Delta z} + \frac{\Delta t}{\varepsilon} \left( \nabla \times \bar{H}^k_{z-\Delta z} \right) \]

\[ m_{E_{z-\Delta z}} = \frac{c_0 \Delta t}{\varepsilon z_{z-\Delta z}} \]

\[ m_{H_{z-\Delta z}} = \frac{c_0 \Delta t}{\mu z_{z-\Delta z}} \]

### Equations → MATLAB Code

**Update Coefficients**

\[ m_{E_{z-\Delta z}} = \frac{c_0 \Delta t}{\varepsilon z_{z-\Delta z}} \quad m_{H_{z-\Delta z}} = \frac{c_0 \Delta t}{\mu z_{z-\Delta z}} \]

\[ m_{E_{z+\Delta z}} = \frac{c_0 \Delta t}{\varepsilon z_{z+\Delta z}} \quad m_{H_{z+\Delta z}} = \frac{c_0 \Delta t}{\mu z_{z+\Delta z}} \]

**Update Equations**

You will need to update the fields at every point in the grid so these equations are placed inside a loop from 1 to Nz.

\[ \bar{H}^k_{z+\Delta z} = \bar{H}^k_{z-\Delta z} + m_{H_{z-\Delta z}} \left( \bar{E}^k_{z+\Delta z} - \bar{E}^k_{z-\Delta z} \right) / \Delta z \]

\[ \bar{E}^k_{z+\Delta z} = \bar{E}^k_{z-\Delta z} + m_{E_{z-\Delta z}} \left( \bar{H}^k_{z+\Delta z} - \bar{H}^k_{z-\Delta z} \right) / \Delta z \]

**% MAIN FDTD LOOP**

for T = 1 : STEPS

% Update H from E
for nz = 1 : Nz

\[ \text{H}(nz) = \text{H}(nz-1) + \text{mHx}(nz) * (\text{E}(nz+1) - \text{E}(nz)) / \Delta z \];

end

% Update E from H
for nz = 1 : Nz

\[ \text{E}(nz) = \text{E}(nz-1) + \text{mEx}(nz) * (\text{H}(nz+1) - \text{H}(nz)) / \Delta z \];

end

end
Each cell has its own update equation and its own update coefficients. They are implemented separately for each cell. All of these equations have the same general form so it is more efficient to implement them using a loop. For a 1D grid with 10 cells, think of it this way...

% Update H from E
Hx(1) = Hx(1) + mHx(1)*(Ey(2) - Ey(1))/dz;
Hx(2) = Hx(2) + mHx(2)*(Ey(3) - Ey(2))/dz;
Hx(3) = Hx(3) + mHx(3)*(Ey(4) - Ey(3))/dz;
Hx(4) = Hx(4) + mHx(4)*(Ey(5) - Ey(4))/dz;
Hx(5) = Hx(5) + mHx(5)*(Ey(6) - Ey(5))/dz;
Hx(6) = Hx(6) + mHx(6)*(Ey(7) - Ey(6))/dz;
Hx(7) = Hx(7) + mHx(7)*(Ey(8) - Ey(7))/dz;
Hx(8) = Hx(8) + mHx(8)*(Ey(9) - Ey(8))/dz;
Hx(9) = Hx(9) + mHx(9)*(Ey(10) - Ey(9))/dz;
Hx(10) = Hx(10) + mHx(10)*(Ey(11) - Ey(10))/dz;

% Update E from H
for nz = 1 : 10
    Hz(nz) = Hz(nz) + mHz(nz)*(Ey(nz+1) - Ey(nz))/dz;
end

% Update H from E
for nz = 1 : 10
    Hx(nz) = Hx(nz) + mHx(nz)*(Ey(nz) - Ey(nz-1))/dz;
end

% Update E from H
for nz = 1 : 10
    Hz(nz) = Hz(nz) + mHz(nz)*(Ez(nz) - Hz(nz-1))/dz;
end

Each cell has its own set of update equations.