Lecture #9

Examples of One-Dimensional FDTD

Lecture Outline

- Review of Lecture 8
  - FDTD Algorithm
  - Code walkthrough
- Simple Electromagnetic Structures
- Two Examples
Review of Lecture #8

Typical FDTD Grid Layout

Note: A real grid would have 200 or more points.
### Initializing the FDTD Simulation

**Initialize Simulation**
- Initialize MATLAB
- Define units
- Define constants

**Define Simulation Parameters**
- Frequency range ($f_{\text{max}}$)
- Device parameters
- Grid parameters (NRES, etc.)

**Compute Grid Resolution**
Initial resolution
$$\Delta' = \min \left( \frac{d_x}{N_x}, \frac{d_y}{N_y}, \frac{d_z}{N_z} \right), \quad N_z \geq 10$$
Snap grid to critical dimension
$$N = \text{ceil} \left( \frac{d_z}{\Delta} \right), \quad \Delta = d_z / N$$

**Build Device on Grid**
Refer to Lecture 3.

**Compute Time Step**
$$\Delta t = \frac{n_v \Delta}{2c_v}$$

**Compute Source**
$$\tau = 0.5 / f_{\text{max}}, \quad t_0 = 6\tau$$
$$E_{src}(t) = \exp \left[ -\frac{(t - t_0)^2}{\tau} \right]$$

**Compute Update Coefficients**
$$m_{ik} = \frac{i_k \Delta}{\mu_z}, \quad m_{ik} = \frac{i_k \Delta}{\varepsilon_z}$$

**Initialize Fields to Zero**
$$F_0 = 0$$

**Initialize Boundary Terms to Zero**
$$h_1 = h_2 = h_3 = e_1 = e_2 = e_3 = 0$$

**Initialize Fourier Transforms**
$$K_z = e^{-j2\pi(kz)}$$

### The Main FDTD Loop

**Update H (Perfectly Absorbing Boundary)**
$$\vec{H}_{k-1}^z = \vec{H}_{k-1}^z + \left( m_{ik} \right) \left( \vec{E}_{k-1}^y - \vec{E}_k^y \right) / \Delta z \quad k < N_z$$

**Handle H Source**
$$E_{src}^z = E_{src}^z - \frac{m_{ik} \vec{E}_{k-1}^y}{\Delta z} \cdot \vec{H}_{k-1}^z$$

**Record H at Boundary**
$$e_1 = e_1, \quad e_2 = e_2, \quad e_3 = E_{\infty}^z$$

**Update E (Perfectly Absorbing Boundary)**
$$\vec{E}_{k-1}^x = \vec{E}_{k-1}^x + \left( m_{ik} \right) \left( \vec{H}_{k-1}^z - \vec{H}_k^z \right) / \Delta z \quad k > 1$$

**Update Fourier Transforms**
$$F_{k+1}^z = F_k^z + \Delta t \cdot (K_z)^T \cdot E_{k+1}^z$$

**Visualize Simulation**
- Superimpose fields on materials
- Show reflectance, transmittance and conservation
- Update only after some number of iterations
The Main FDTD Loop (Pseudo Code)

```matlab
% MAIN FDTD LOOP
for T = 1 : STEPS
    % Update H from E
    for nz = 1 : Nz
        Update Hx(nz)
    end
    % H Source
    Correct Hx(nz_src-1)
    % Record H-Field at Boundary
    H1 <-> Hx(1)
    % Update E from H
    for nz = 1 : Nz
        Update Ey(nz)
    end
    % E Source
    Correct Ey(nz_src)
    % Record E-Field Boundary
    E1 <-> Ey(Nz)
    % Update Fourier Transforms
    for nf = 1 : NFREQ
        Integrate REF(nf), TRN(nf), and SRC(nf)
    end
    % Visualize
    Plot fields, materials, and response
end
```

Post Processing

- **Compute Response**
  - \( R(f) = \left( \frac{F_{ref}(f)}{\text{FFT}[E_m(t)]} \right)^2 \)
  - \( T(f) = \left( \frac{F_{trn}(f)}{\text{FFT}[E_m(t)]} \right)^2 \)
  - \( C(f) = R(f) \times T(f) \)

- **Visualize Results**
  - Superimpose fields on materials
  - Show reflectance, transmittance and conservation
  - Show response on linear and dB scale

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Lecture 9  Slide 7
Lecture 9  Slide 8
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Outline of Steps for FDTD Analysis

• Step 1: Define problem
  – What device are you modeling?
  – What is its geometry?
  – What materials is it made of?
  – What do you want to learn about the device?

• Step 2: Initialize FDTD
  – Compute grid resolution
  – Assign materials values to points on the grid
  – Compute time step
  – Initialize Fourier transforms

• Step 3: Run FDTD
• Step 4: Analyze the data

Step 1: Define the Problem

What device are you modeling? – A dielectric slab
What is its geometry? – 1 foot thick slab
What materials it is made from? – \( \mu_r = 2.0, \varepsilon_r = 6.0 \) (outside is air)
What do you want to learn? – reflectance and transmittance from 0 to 1 GHz

1.0 ft
\[ \mu_r = 2.0, \varepsilon_r = 6.0 \]
Air

1.0 ft
Air
### Step 2: Compute Grid (1 of 2)

#### Initial Grid Resolution (Wavelength)

\[ N_x = 20 \]

\[ n_{\text{max}} = \sqrt{\mu_r \varepsilon_r} = \sqrt{(2.0)(6.0)} = 3.46 \]

\[ \lambda_{\text{min}} = \frac{c_0}{f_{\text{max}} n_{\text{max}}} = \frac{299792458}{(1.0 \text{ GHz})(3.46)} = 8.6543 \text{ cm} \]

\[ \Delta_x = \frac{\lambda_{\text{min}}}{N_x} = \frac{8.6543 \text{ cm}}{20} = 0.4327 \text{ cm} \]

#### Initial Grid Resolution (Structure)

\[ N_y = 4 \]

\[ \Delta_y = \frac{d}{N_y} = \frac{30.48 \text{ cm}}{4} = 7.6200 \text{ cm} \]

#### Initial Grid Resolution (Overall)

\[ \Delta z' = \min(\Delta_x, \Delta_y) = 0.4327 \text{ cm} \]

### Step 2: Compute Grid (2 of 2)

#### Snap Grid to Critical Dimension(s)

The number of grid cells representing the thickness of the dielectric slab is

\[ N' = \frac{d_z}{\Delta z'} = \frac{30.48 \text{ cm}}{0.4327 \text{ cm}} = 70.44 \text{ cells} \]

It is impossible to represent the thickness of the slab exactly with this grid resolution.

To represent the thickness of the slab exactly, we round \( N' \) up to the nearest integer and then calculate the grid resolution based on this quantity.

\[ N = \text{round up} \left[ N' \right] = 71 \text{ cells} \]

\[ \Delta z = \frac{d_z}{N} = \frac{30.48 \text{ cm}}{71} = 0.4293 \text{ cm} \]
Step 2: Build Device on the Grid (1 of 2)

Determine Size of Grid

We need to have enough grid cells to fit the device being modeled, some space on either side of the device (10 cells for now), and cells for injecting the source and recording transmitted and reflected fields.

$$N_z = 71 + 2(10 \text{ cells}) + 3 = 94 \text{ cells}$$

Step 2: Build Device on the Grid (2 of 2)

Compute Position of Materials on Grid

$$n_{z,1} = 2 + 10 + 1 = 13$$
$$n_{z,2} = n_{z,1} + \text{round}[d/\Delta z] - 1 = 13 + 71 - 1 = 83$$

Add Materials to Grid

$$UR(nz1:nz2) = ur;$$
$$ER(nz1:nz2) = er;$$
Step 2: Initialize FDTD (1 of 2)

Compute the Time Step
\[ \Delta t = \frac{n_0 \Delta z}{2c_0} = \frac{(1.0)(0.4293 \text{ cm})}{2(299792458 \text{ cm/s})} = 7.1599 \times 10^{-12} \text{ sec} \]

Compute Source Parameters
\[ \tau = \frac{1}{2 f_{\text{max}}} = \frac{1}{2(1 \text{ GHz})} = 5.00 \times 10^{-10} \text{ sec} \]
\[ t_0 = 6\tau = 3.00 \times 10^{-9} \text{ sec} \]

Compute Number of Time Steps
\[ t_{\text{prop}} = \frac{n_p N_{\text{prop}} \Delta z}{c_0} = \frac{(3.46)(94)(0.4293 \text{ cm})}{299792458 \text{ cm/s}} = 4.6629 \times 10^{-9} \text{ sec} \]
\[ T = 12\tau + 6 t_{\text{prop}} = 12 \left( 5 \times 10^{-10} \text{ s} \right) + 5 \left( 4.6597 \times 10^{-9} \text{ s} \right) = 2.9314 \times 10^{-8} \text{ sec} \]
\[ \text{STEPS} = \text{round} \left( \frac{T}{\Delta t} \right) = 4095 \]

Step 2: Initialize FDTD (2 of 2)

Compute the Source Functions for Ey/Hx Mode
\[ \delta t = \frac{n_{\text{src}} \Delta z}{2c_0} + \frac{\Delta t}{2} = 1.0740 \times 10^{-11} \text{ sec} \]
\[ A = -\sqrt{\frac{\varepsilon_{\text{src}}}{\mu_{\text{src}}}} = -\sqrt{\frac{1.0}{1.0}} = 1 \]
\[ E_y(t) = \exp \left[ -\left( \frac{t - t_0}{\tau} \right)^2 \right] \]
\[ \tilde{H}_x(t) = A \exp \left[ -\left( \frac{t - t_0 + \delta t}{\tau} \right)^2 \right] \]

% COMPUTE GAUSSIAN SOURCE FUNCTIONS
% time axis
% total delay between E and H
% amplitude of H field
% field source

Initialize the Fourier Transforms
% INITIALIZE FOURIER TRANSFORMS
NFREQ = 100;
FREQ = linspace(0,1*gigahertz,NFREQ);
K = exp(-i*2*pi*dt*FREQ);
REF = zeros(1,NFREQ);
TRN = zeros(1,NFREQ);
SRC = zeros(1,NFREQ);
Step 3: Run FDTD (3 of 3)

[Diagram showing the process of running FDTD, including steps for handling H-field source, E-field source, and updating Fourier transforms, with visualizations of the process steps.]

Step 4: Analyze the Data

Normalize the Data to the Source Spectrum

\[ R(f) = \left( \frac{F_{\text{ref}}(f)}{\text{FFT}\left[E_{\text{src}}(t)\right]} \right)^2 \]

\[ T(f) = \left( \frac{F_{\text{trn}}(f)}{\text{FFT}\left[E_{\text{src}}(t)\right]} \right)^2 \]

\[ C(f) = R(f) + T(f) \]

% COMPUTE REFLECTANCE
% AND TRANSMITTANCE
REF = abs(REF./SRC).^2;
TRN = abs(TRN./SRC).^2;
CON = REF + TRN;

[Graph showing reflectance and transmittance data across frequency (GHz).]
Reflection and Transmission at an Interface

Reflection and Transmission Coefficients
At normal incidence, the field amplitude of waves reflected from, or transmitted through, an interface are related to the incident wave through the reflection and transmission coefficients.

\[ r = \frac{\eta_2 - \eta_i}{\eta_2 + \eta_i} \]
\[ t = \frac{2\eta_2}{\eta_2 + \eta_i} \]

Reflectance and Transmittance
The reflectance and transmittance quantify the fraction of power that is reflected from, or transmitted through, an interface.

\[ R = |r|^2 \]
\[ T = |t|^2 \]

Useful Special Cases
For \( \epsilon_r = 9.0 \) and \( \mu_r = 1.0 \), \( R = 25\% \) and \( T = 75\% \).
For \( \epsilon_r = 1.0 \) and \( \mu_r = 9.0 \), \( R = 25\% \) and \( T = 75\% \).
For \( \epsilon_r = 9.0 \) and \( \mu_r = 9.0 \), \( R = 0\% \) and \( T = 100\% \).
**Anti-Reflection Layer**

- \( n_1, \eta_1 \) to \( n_2, \eta_2 \)

\[ n_{ar} = \sqrt{\eta_1 \eta_2} \]

General Case

\[ L = \frac{\lambda_0}{4n_{ar}} \]

No magnetic response

**Bragg Gratings**

- Each layer is \( \lambda/4 \) thick.
- Higher index contrast provides wider stop band.
- More layers improve suppression in the stop band.

\[ L_L = \frac{\lambda_0}{4n_L} \]

\[ L_H = \frac{\lambda_0}{4n_H} \]
Example #1: The Invisible Slab

Design Problem

A radome is being designed to protect an antenna operating at 2.4 GHz. For mechanical reasons, it must be constructed from 1 ft thick plastic with dielectric constant 12. How could you modify the design to maximize transmission through the radome? Simulate the design using 1D FDTD.
A Solution

Add anti-reflection layers to both sides of the radome.

![Diagram of radome with anti-reflection layers]

The Design

To match the slab material to air on both sides, the dielectric constant and thickness of the anti-reflection layers should be

\[ \varepsilon_2 = \sqrt{\varepsilon_1 \varepsilon_{\text{air}}} = \sqrt{12 \times 1} = 3.46 \]
\[ n_2 = \sqrt{\varepsilon_2} = \sqrt{3.46} = 1.86 \]
\[ \lambda_0 = \frac{c_0}{f_0} = \frac{299792458 \ \text{m/s}}{2.4 \times 10^9 \ \text{Hz}} = 12.49 \ \text{cm} \]
\[ d_2 = \frac{\lambda_0}{4n_2} = \frac{12.49 \ \text{cm}}{4(1.86)} = 1.6779 \ \text{cm} \]
Example #2: The Blinded Missile
Design Problem

A heat-seeking missile is vulnerable to jamming from high power lasers operating at $\lambda_0=980$ nm. Design a multilayer cover that would prevent this energy from reaching the infrared camera. The design should provide at least 30 dB of suppression at 980 nm. Simulate the design using 1D FDTD. The only materials available to you are SiO$_2$ ($n_{\text{SiO}_2} = 1.5$) and SiN ($n_{\text{SiN}} = 2.0$).

A Solution

Use a Bragg grating with alternating layers of SiO$_2$ and SiN.

$n_{\text{SiO}_2} = 1.5$
$n_{\text{SiN}} = 2.0$
The Design

\[ d_1 = \frac{\lambda_0}{4n_1} = \frac{980 \text{ nm}}{4(1.5)} = 163 \text{ nm} \]
\[ d_2 = \frac{\lambda_0}{4n_2} = \frac{980 \text{ nm}}{4(2.0)} = 122 \text{ nm} \]

But how many layers?

Number of Layers for 30 dB Suppression

10 periods, barely 20 dB 😞
20 periods, 45 dB! Yikes!!
15 periods, 30 dB. 😊

In practice, you may want to include a few extra layers as a safety margin.
Manufacturing inaccuracies often degrade performance.
FDTD Simulation Results

Figure 1

FIELD AT STEP 34000 OF 34023

REFLECTANCE AND TRANSMITTANCE

Wavelength (nm)