ELECTROSTATIC FIELDS AND CHARGES

EE 3321 Electromagnetic Field Theory

Pioneering 21st Century Electromagnetics and Photonics

http://emlab.utep.edu

GOVERNING EQS.

Conditions:

1. $\omega = 0$.
2. Size << λ.

$$\nabla \times \vec{E} = 0$$

$$\nabla \bullet \vec{D} = \rho_v$$

$$\vec{D} = [\varepsilon]\vec{E}$$

POWER AND ENERGY

Energy in the Field

$$W = \begin{cases} \frac{1}{2}\iiint_v (\vec{D} \bullet \vec{E})\, dv & \text{general case} \\ \frac{1}{2}\iiint_v \varepsilon |\vec{E}|^2\, dv & \text{LHI case} \end{cases}$$

ELECTRIC POTENTIAL

Calculating E from V:

$$\vec{E} = -\nabla V$$

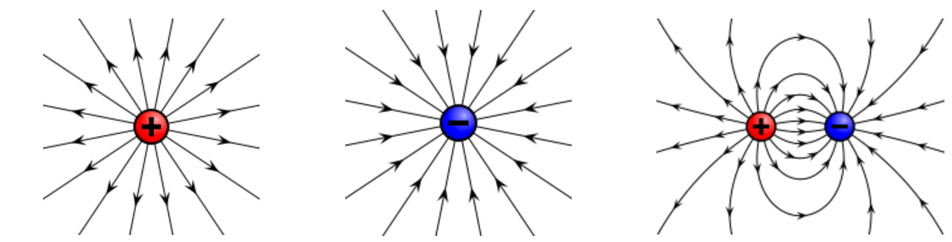
Calculating V from E:

$$V_{AB} = V_B - V_A = \frac{W}{Q} = -\int_A^B \vec{E} \bullet d\vec{\ell}$$

Power in Conductors

$$P = \begin{cases} \iiint_V \vec{E} \bullet \vec{J} dv & \text{General Joule's Law} \\ \iiint_V \sigma |\vec{E}|^2\, dv & \text{More common form} \end{cases}$$

POINT CHARGES



Single Charge:

$$\vec{D} = \frac{Q}{4\pi}\frac{\hat{a}_R}{R^2} = \frac{Q \cdot (\vec{r} - \vec{r}_Q)}{4\pi |\vec{r} - \vec{r}_Q|^3}$$

$$\vec{E} = \vec{D}/\varepsilon_0 \varepsilon_r$$

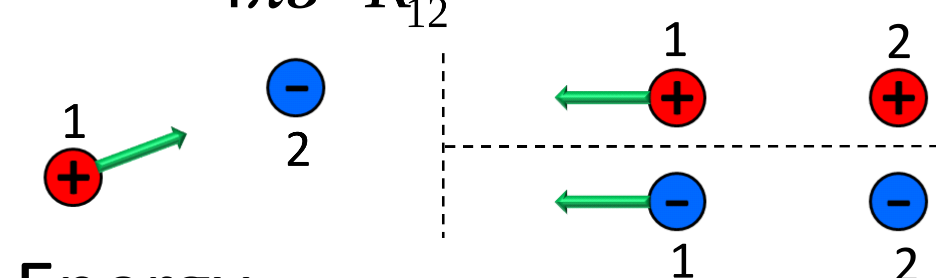
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\frac{Q_i}{|\vec{r} - \vec{r}_i|}$$

Force:

$$\vec{F} = Q\vec{E}$$

Coulomb's Law:

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon}\frac{\hat{a}_{12}}{R_{12}^2} \qquad \vec{R}_{12} = \vec{r}_2 - \vec{r}_1$$



Energy:

$$W = \frac{1}{2}\sum_{i=1}^{N} Q_i V_i$$

CHARGE DISTRIBUTIONS

Qty.	LINE, ρ_L (C/m)	SURFACE, ρ_s (C/m²)	VOLUME, ρ_v (C/m³)
Total Charge	$Q = \int_L \rho_L d\ell$	$Q = \iint_S \rho_S ds$	$Q = \iiint_v \rho_v dv$
Total Field	$\vec{D} = \int_L \frac{\rho_L d\ell}{4\pi R^2}\hat{a}_R$	$\vec{D} = \iint_S \frac{\rho_S ds}{4\pi R^2}\hat{a}_R$	$\vec{D} = \iiint_v \frac{\rho_v dv}{4\pi R^2}\hat{a}_R$
Electric Potential	$V = \frac{1}{4\pi\varepsilon}\int_L \frac{\rho_L(\vec{r}')d\ell'}{\lvert\vec{r} - \vec{r}'\rvert}$	$V = \frac{1}{4\pi\varepsilon}\iint_S \frac{\rho_S(\vec{r}')ds'}{\lvert\vec{r} - \vec{r}'\rvert}$	$V = \frac{1}{4\pi\varepsilon}\iiint_v \frac{\rho_v(\vec{r}')dv'}{\lvert\vec{r} - \vec{r}'\rvert}$
Energy	$W = \frac{1}{2}\int_L \rho_L V d\ell$	$W = \frac{1}{2}\iint_S \rho_s V ds$	$W = \frac{1}{2}\iiint_v \rho_v V dv$

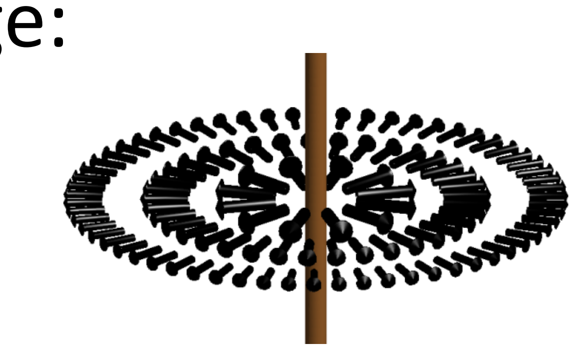
Uniform and Finite Line Charge:

$$\vec{D}_{\text{total}} = \frac{\rho_L}{4\pi\rho}\left[(\sin\alpha_1 - \sin\alpha_2)\hat{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\hat{a}_z\right]$$

$$Q_{\text{total}} = \rho_L L$$

Uniform and Infinite Line Charge:

$$Q_{\text{total}} = \infty \qquad \vec{D}_{\text{total}} = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho$$



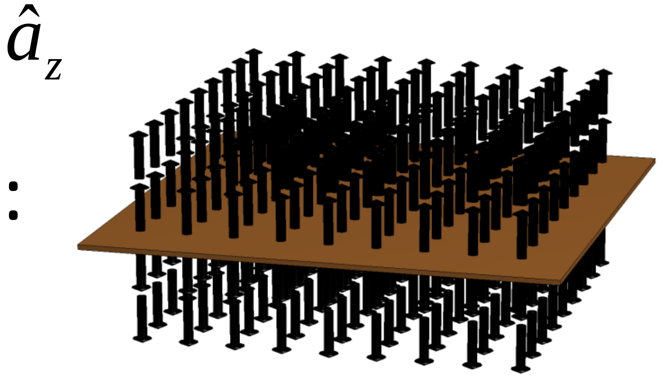
Uniform Circular Plate Charge:

$$Q_{\text{total}} = \pi R^2 \rho_S$$

$$\vec{D}_{\text{total}}(0,0,z) = \frac{\rho_s}{2}\left(1 - \frac{z}{\sqrt{R^2 + z^2}}\right)\hat{a}_z$$

Uniform Infinite Sheet Charge:

$$Q_{\text{total}} = \infty \qquad \vec{D}_{\text{total}} = \frac{\rho_S}{2}\hat{a}_z$$



Uniform Spherical Charge → Like a point charge.