MAGNETIC FIELDS AND CURRENTS

EE 3321 Electromagnetic Field Theory

Pioneering 21st Century Electromagnetics and Photonics

http://emlab.utep.edu

MAXWELL'S EQS.

Conditions:

1. $\omega = 0$.
2. Size << λ.

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \bullet \vec{B} = 0$$

$$\vec{B} = [\mu]\vec{H}$$

LORENTZ FORCE EQ.

$$\vec{F} = Q\vec{E} + Q\vec{u} \times \vec{B}$$

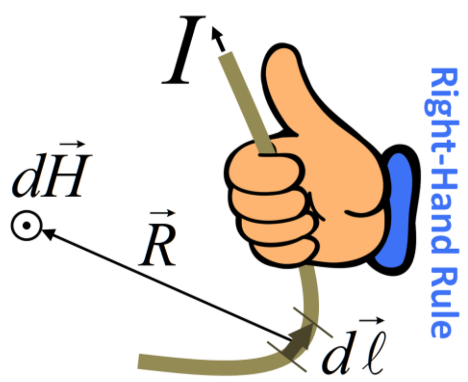
$Q\vec{E}$: Force due to electric field

$Q\vec{u} \times \vec{B}$: Force due to magnetic field

MAGNETIC FIELDS AROUND CURRENT ELEMENTS

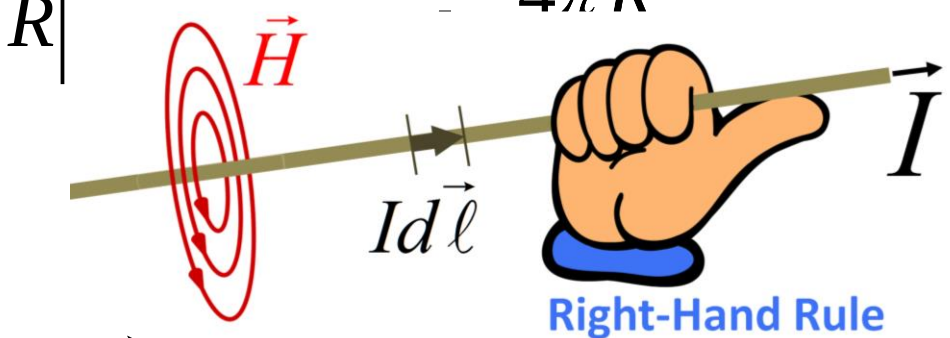
Biot-Savart Law

$$d\vec{H} = \frac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2} = \frac{Id\vec{\ell} \times \vec{R}}{4\pi \left|\vec{R}\right|^3}$$



Line Current

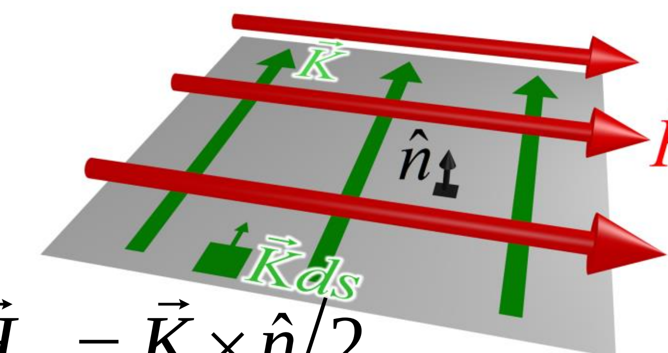
$$\vec{H} = \int \frac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2}$$



$$\vec{H}_\infty = \left(I/2\pi\rho\right)\hat{a}_\phi$$

Surface Current

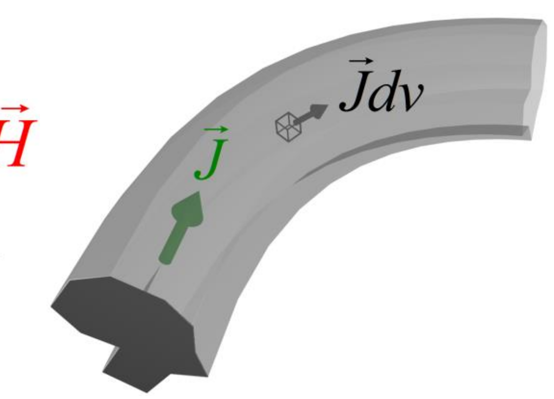
$$\vec{H} = \iint_S \frac{\vec{K}ds \times \hat{a}_R}{4\pi R^2}$$



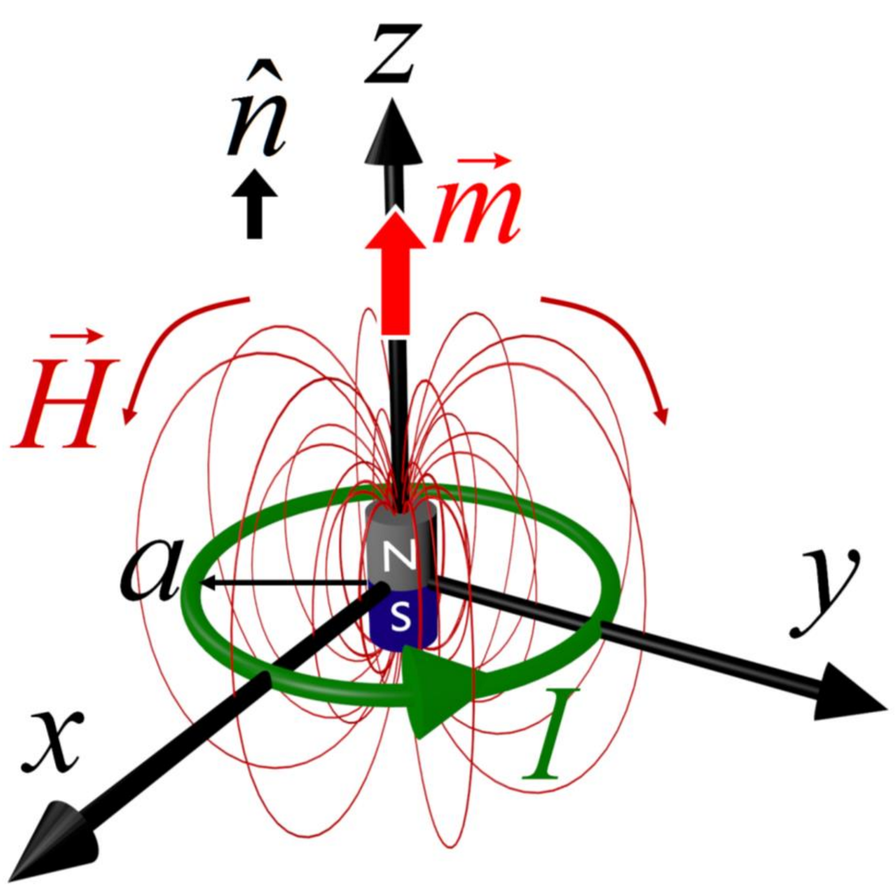
$$\vec{H}_\infty = \vec{K} \times \hat{n}/2$$

Volume Current

$$\vec{H} = \iiint_V \frac{\vec{J}dv \times \hat{a}_R}{4\pi R^2}$$



MAGNETIC DIPOLE



Magnetic Dipole Moment

$$\vec{m} = \pi a^2 I\hat{n}$$

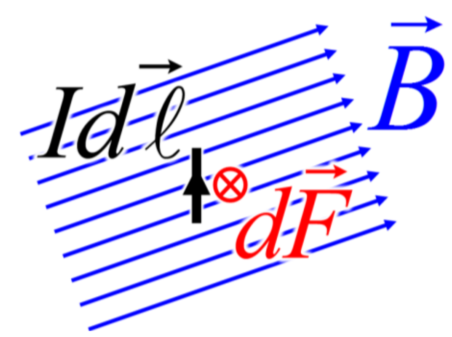
Magnetic Field

$$\vec{H} = \frac{|\vec{m}|}{4\pi r^2}\left(2\cos\theta\hat{a}_r + \sin\theta\hat{a}_\theta\right)$$

FORCE THAT MAGNETIC FIELDS PUT ON CURRENT ELEMENTS

Differential Current

$$d\vec{F} = Id\vec{\ell} \times \vec{B}$$



Line Current

$$\vec{F} = \int_L Id\vec{\ell} \times \vec{B}$$

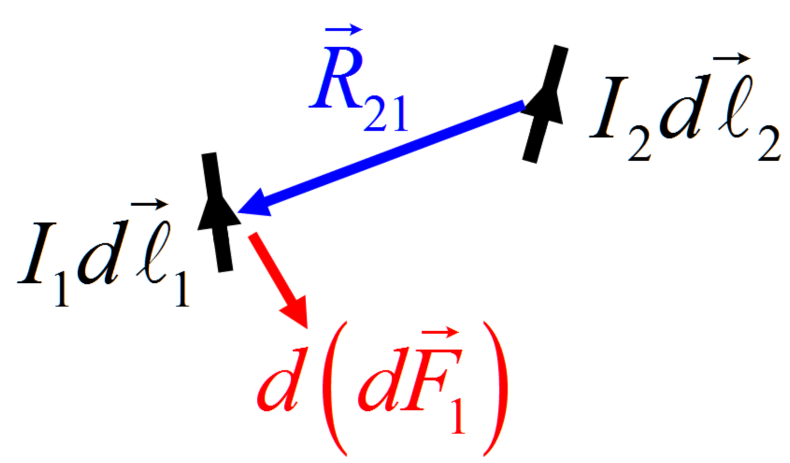
Surface Current

$$\vec{F} = \iint_S \vec{K}ds \times \vec{B}$$

Volume Current

$$\vec{F} = \iiint_V \vec{J}dv \times \vec{B}$$

FORCE BETWEEN TWO CURRENT ELEMENTS



$$d\left(d\vec{F}_1\right) = \frac{\mu}{4\pi}\frac{\left(I_1 d\vec{\ell}_1\right) \times \left(I_2 d\vec{\ell}_2\right) \times \hat{a}_{21}}{R_{21}^2}$$

$$\vec{F}_1 = \int_{L1}\int_{L2} d\left(d\vec{F}_1\right) = \frac{\mu I_1 I_2}{4\pi}\int_{L1}\int_{L2}\frac{d\vec{\ell}_1 \times d\vec{\ell}_2 \times \hat{a}_{21}}{R_{21}^2}$$

MAGNETIC TORQUE & MOMENT

Magnetic Dipole Moment

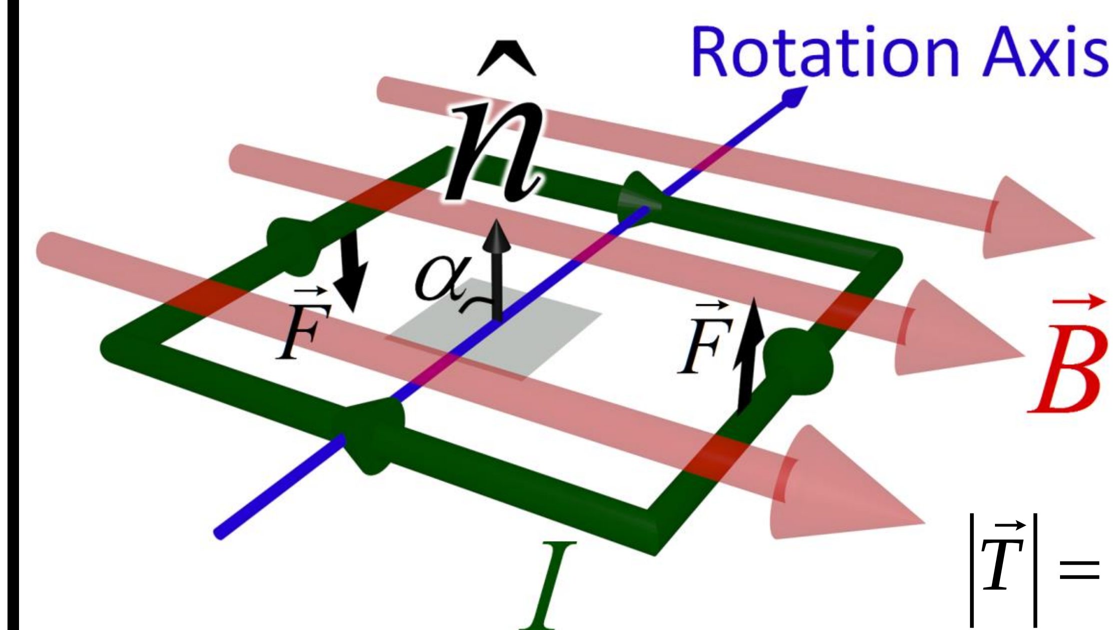
$$\vec{m} = SI\hat{n} \quad (\text{A/m})$$

$S \equiv$ area of loop

Torque

$$\vec{T} = \vec{m} \times \vec{B}$$

Acts to reduce α and put m and B in same direction.



$$\left|\vec{T}\right| = BIS\sin\alpha$$