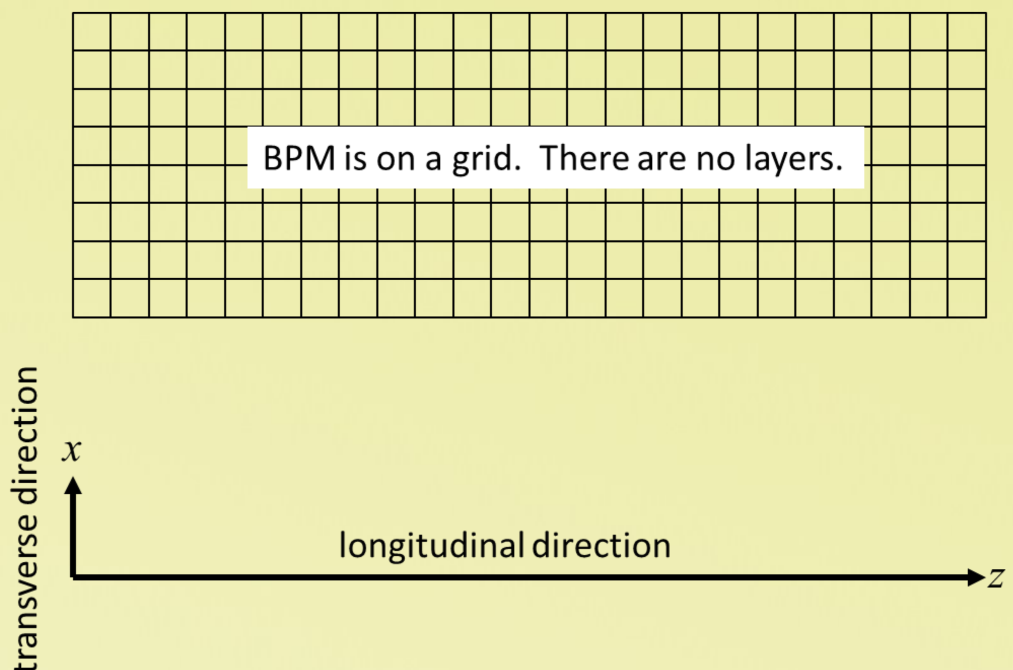
THE UNIVERSITY OF TEXAS AT EL PASO

Beam Propagation Method

The beam propagation method (BPM) is widely used in photonics and nonlinear optics. It propagates a beam through nonhomogeneous media and achieves its efficiency by handling the propagation problem one grid slice at a time. The basis formulation assumes one-way propagation under the paraxial approximation. Bi-directional and wide-angle formulations exist.

GEOMETRY OF THE BEAM PROPAGATION METHOD



BPM is primarily a "forward" propagating algorithm where the dominant direction of propagation is longitudinal.

The grid is computed and interpreted as it is in FDFD. The algorithm and implementation looks more like the method of lines.

BENEFITS

- Highly efficient method
- Can easily incorporate nonlinear materials properties. This is very unique for a frequency-domain method.
- Simple to formulate and implement
- FFT-BPM is simpler to formulate and implement.
- BPM is commonly used to model nonlinear optical devices and waveguide circuits.

DRAWBACKS

- Not a rigorous method
- Small angle approximation
- Ignores backward reflections
- FFT-BPM is slower, less stable, and less versatile than FD-BPM

FORMULATION OF THE BASIC BPM ALGORITHM

Step 1: Start with Maxwell's equations.

$$\frac{\partial E_z}{\partial y'}-\frac{\partial E_y}{\partial z'}=\mu'_{xx}\tilde{H}_x \qquad \frac{\partial \tilde{H}_z}{\partial y'}-\frac{\partial \tilde{H}_y}{\partial z'}=\varepsilon'_{xx}E_x \qquad x'=k_0x \qquad \varepsilon'_{xx}=\varepsilon_{xx}\frac{s_y}{s_x} \qquad \varepsilon'_{yy}=\varepsilon_{yy}\frac{s_x}{s_y} \qquad \varepsilon'_{zz}=\varepsilon_{zz}s_xs_y$$

$$\frac{\partial E_x}{\partial z'}-\frac{\partial E_z}{\partial x'}=\mu'_{yy}\tilde{H}_y \qquad \frac{\partial \tilde{H}_x}{\partial z'}-\frac{\partial \tilde{H}_z}{\partial x'}=\varepsilon'_{yy}E_y \qquad y'=k_0y$$

$$\frac{\partial E_y}{\partial x'}-\frac{\partial E_x}{\partial y'}=\mu'_{zz}\tilde{H}_z \qquad \frac{\partial \tilde{H}_y}{\partial x'}-\frac{\partial \tilde{H}_x}{\partial y'}=\varepsilon'_{zz}E_z \qquad z'=k_0z \qquad \mu'_{xx}=\mu_{xx}\frac{s_y}{s_x} \qquad \mu'_{yy}=\mu_{yy}\frac{s_x}{s_y} \qquad \mu'_{zz}=\mu_{zz}s_xs_y$$

Step 2: Reduce problem to two dimensions. $\frac{\partial}{\partial y'}=0$

$$-\frac{\partial E_y}{\partial z'}=\mu'_{xx}\tilde{H}_x \qquad -\frac{\partial \tilde{H}_y}{\partial z'}=\varepsilon'_{xx}E_x$$

$$\frac{\partial E_x}{\partial z'}-\frac{\partial E_z}{\partial x'}=\mu'_{yy}\tilde{H}_y \qquad \frac{\partial \tilde{H}_x}{\partial z'}-\frac{\partial \tilde{H}_z}{\partial x'}=\varepsilon'_{yy}E_y$$

$$\frac{\partial E_y}{\partial x'}=\mu'_{zz}\tilde{H}_z \qquad \frac{\partial \tilde{H}_y}{\partial x'}=\varepsilon'_{zz}E_z$$

E-Mode

$$\frac{\partial \tilde{H}_x}{\partial z'}-\frac{\partial \tilde{H}_z}{\partial x'}=\varepsilon'_{yy}E_y \qquad -\frac{\partial E_y}{\partial z'}=\mu'_{xx}\tilde{H}_x \qquad \frac{\partial E_y}{\partial x'}=\mu'_{zz}\tilde{H}_z$$

H-Mode

$$\frac{\partial E_x}{\partial z'}-\frac{\partial E_z}{\partial x'}=\mu'_{yy}\tilde{H}_y \qquad -\frac{\partial \tilde{H}_y}{\partial z'}=\varepsilon'_{xx}E_x \qquad \frac{\partial \tilde{H}_y}{\partial x'}=\varepsilon'_{zz}E_z$$

Step 3: Assume a solution using the slowly varying envelope approximation.

$$\vec{E}(x,z)\approx\vec{\xi}(x,z)e^{jn_{\text{eff}}z'}$$
$$\vec{H}(x,z)\approx\vec{\psi}(x,z)e^{jn_{\text{eff}}z'}$$

E-Mode

$$jn_{\text{eff}}\psi_x+\frac{\partial\psi_x}{\partial z'}-\frac{\partial\psi_z}{\partial x'}=\varepsilon'_{yy}\xi_y \qquad -jn_{\text{eff}}\xi_y-\frac{\partial\xi_y}{\partial z'}=\mu'_{xx}\psi_x \qquad \frac{\partial\xi_y}{\partial x'}=\mu'_{zz}\psi_z$$

H-Mode

$$jn_{\text{eff}}\xi_x+\frac{\partial\xi_x}{\partial z'}-\frac{\partial\xi_z}{\partial x'}=\mu'_{yy}\psi_y \qquad -jn_{\text{eff}}\psi_y-\frac{\partial\psi_y}{\partial z'}=\varepsilon'_{xx}\xi_x \qquad \frac{\partial\psi_y}{\partial x'}=\varepsilon'_{zz}\xi_z$$

Step 4: Write equations in matrix form.

E-Mode

$$jn_{\text{eff}}\mathbf{h}_x+\frac{\partial\mathbf{h}_x}{\partial z'}-\mathbf{D}_x^h\mathbf{h}_z=\boldsymbol{\varepsilon}_{yy}\mathbf{e}_y \qquad -jn_{\text{eff}}\mathbf{e}_y-\frac{\partial\mathbf{e}_y}{\partial z'}=\boldsymbol{\mu}_{xx}\mathbf{h}_x \qquad \mathbf{D}_x^e\mathbf{e}_y=\boldsymbol{\mu}_{zz}\mathbf{h}_z$$

H-Mode

$$jn_{\text{eff}}\mathbf{e}_x+\frac{\partial\mathbf{e}_x}{\partial z'}-\mathbf{D}_x^e\mathbf{e}_z=\boldsymbol{\mu}_{yy}\mathbf{h}_y \qquad -jn_{\text{eff}}\mathbf{h}_y-\frac{\partial\mathbf{h}_y}{\partial z'}=\boldsymbol{\varepsilon}_{xx}\mathbf{e}_x \qquad \mathbf{D}_x^h\mathbf{h}_y=\boldsymbol{\varepsilon}_{zz}\mathbf{e}_z$$

Step 5: Derive matrix wave equation with small angle approximation.

E-Mode

$$\cancel{\frac{\partial^2\mathbf{e}_y}{\partial z'^2}}+j2n_{\text{eff}}\frac{\partial\mathbf{e}_y}{\partial z'}+\boldsymbol{\mu}_{xx}\mathbf{D}_x^H\boldsymbol{\mu}_{zz}^{-1}\mathbf{D}_x^E\mathbf{e}_y+\left(\boldsymbol{\mu}_{xx}\boldsymbol{\varepsilon}_{yy}-n_{\text{eff}}^2\mathbf{I}\right)\mathbf{e}_y=\mathbf{0} \qquad \frac{\partial\mathbf{e}_y}{\partial z'}=\frac{j}{2n_{\text{eff}}}\left(\boldsymbol{\mu}_{xx}\mathbf{D}_x^H\boldsymbol{\mu}_{zz}^{-1}\mathbf{D}_x^E+\boldsymbol{\mu}_{xx}\boldsymbol{\varepsilon}_{yy}-n_{\text{eff}}^2\mathbf{I}\right)\mathbf{e}_y$$

H-Mode

$$\cancel{\frac{\partial^2\mathbf{h}_y}{\partial z'^2}}+j2n_{\text{eff}}\frac{\partial\mathbf{h}_y}{\partial z'}+\boldsymbol{\varepsilon}_{xx}\mathbf{D}_x^e\boldsymbol{\varepsilon}_{zz}^{-1}\mathbf{D}_x^h\mathbf{h}_y+\left(\boldsymbol{\varepsilon}_{xx}\boldsymbol{\mu}_{yy}-n_{\text{eff}}^2\mathbf{I}\right)\mathbf{h}_y=\mathbf{0} \qquad \frac{\partial\mathbf{h}_y}{\partial z'}=\frac{j}{2n_{\text{eff}}}\left(\boldsymbol{\varepsilon}_{xx}\mathbf{D}_x^e\boldsymbol{\varepsilon}_{zz}^{-1}\mathbf{D}_x^h+\boldsymbol{\varepsilon}_{xx}\boldsymbol{\mu}_{yy}-n_{\text{eff}}^2\mathbf{I}\right)\mathbf{h}_y$$

Step 6: Approximate the z-derivative

E-Mode

$$\frac{\mathbf{e}_y^{i+1}-\mathbf{e}_y^i}{\Delta z'}=\frac{j}{2n_{\text{eff}}}\frac{\mathbf{A}_e^{i+1}\mathbf{e}_y^{i+1}+\mathbf{A}_e^i\mathbf{e}_y^i}{2} \qquad \mathbf{A}_e^i=\boldsymbol{\mu}_{xx}^i\mathbf{D}_x^H\left(\boldsymbol{\mu}_{zz}^i\right)^{-1}\mathbf{D}_x^E+\boldsymbol{\mu}_{xx}^i\boldsymbol{\varepsilon}_{yy}^i-n_{\text{eff}}^2\mathbf{I}$$

H-Mode

$$\frac{\mathbf{h}_y^{i+1}-\mathbf{h}_y^i}{\Delta z'}=\frac{j}{2n_{\text{eff}}}\frac{\mathbf{A}_h^{i+1}\mathbf{h}_y^{i+1}+\mathbf{A}_h^i\mathbf{h}_y^i}{2} \qquad \mathbf{A}_h^i=\boldsymbol{\varepsilon}_{xx}^i\mathbf{D}_x^h\left(\boldsymbol{\varepsilon}_{zz}^i\right)^{-1}\mathbf{D}_x^e+\boldsymbol{\varepsilon}_{xx}^i\boldsymbol{\mu}_{yy}^i-n_{\text{eff}}^2\mathbf{I}$$

Step 7: Solve field at $i+1$

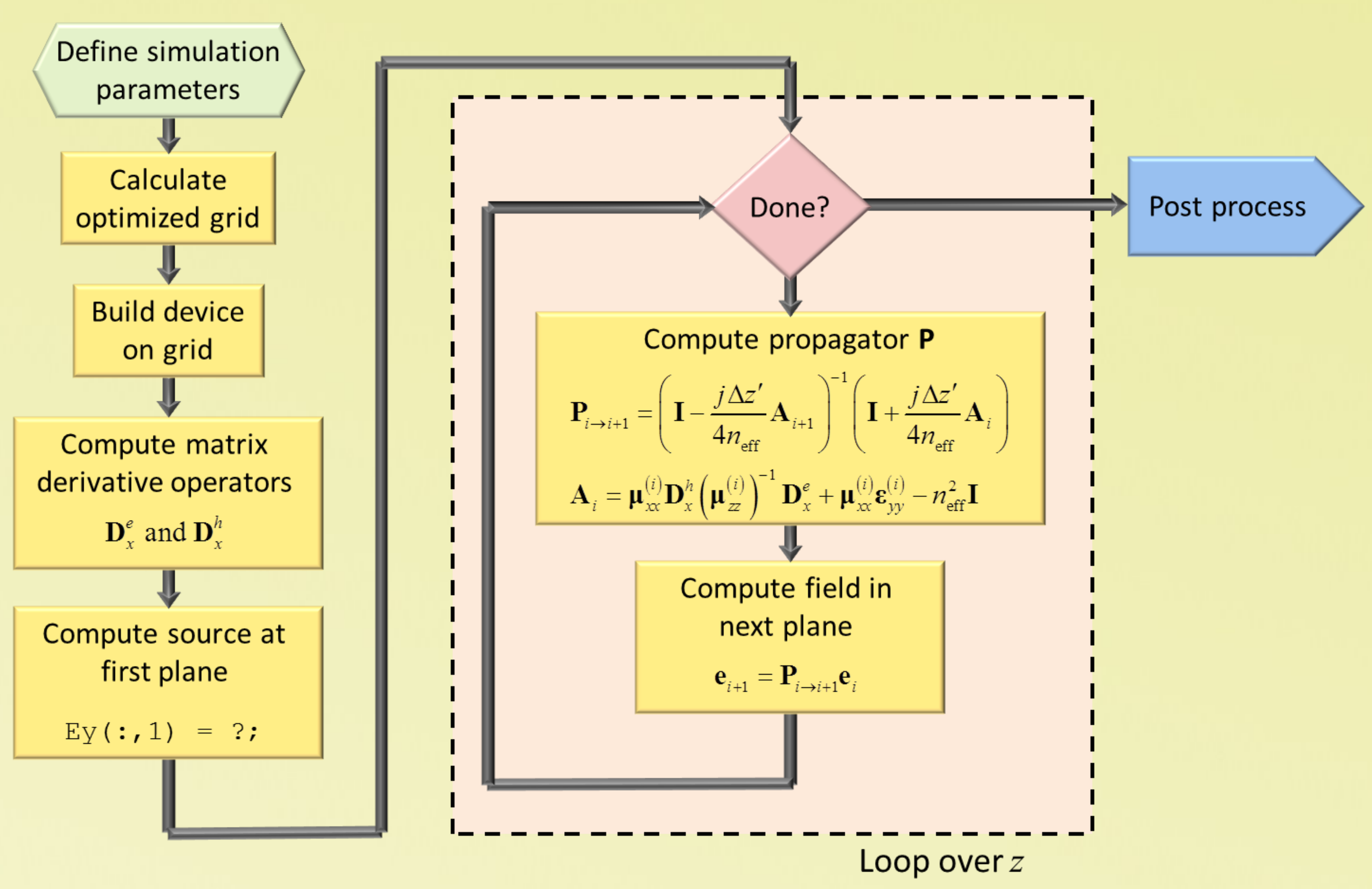
E-Mode

$$\mathbf{e}_y^{i+1}=\left(\mathbf{I}-\frac{j\Delta z'}{4n_{\text{eff}}}\mathbf{A}_e^{i+1}\right)^{-1}\left(\mathbf{I}+\frac{j\Delta z'}{4n_{\text{eff}}}\mathbf{A}_e^i\right)\mathbf{e}_y^i$$

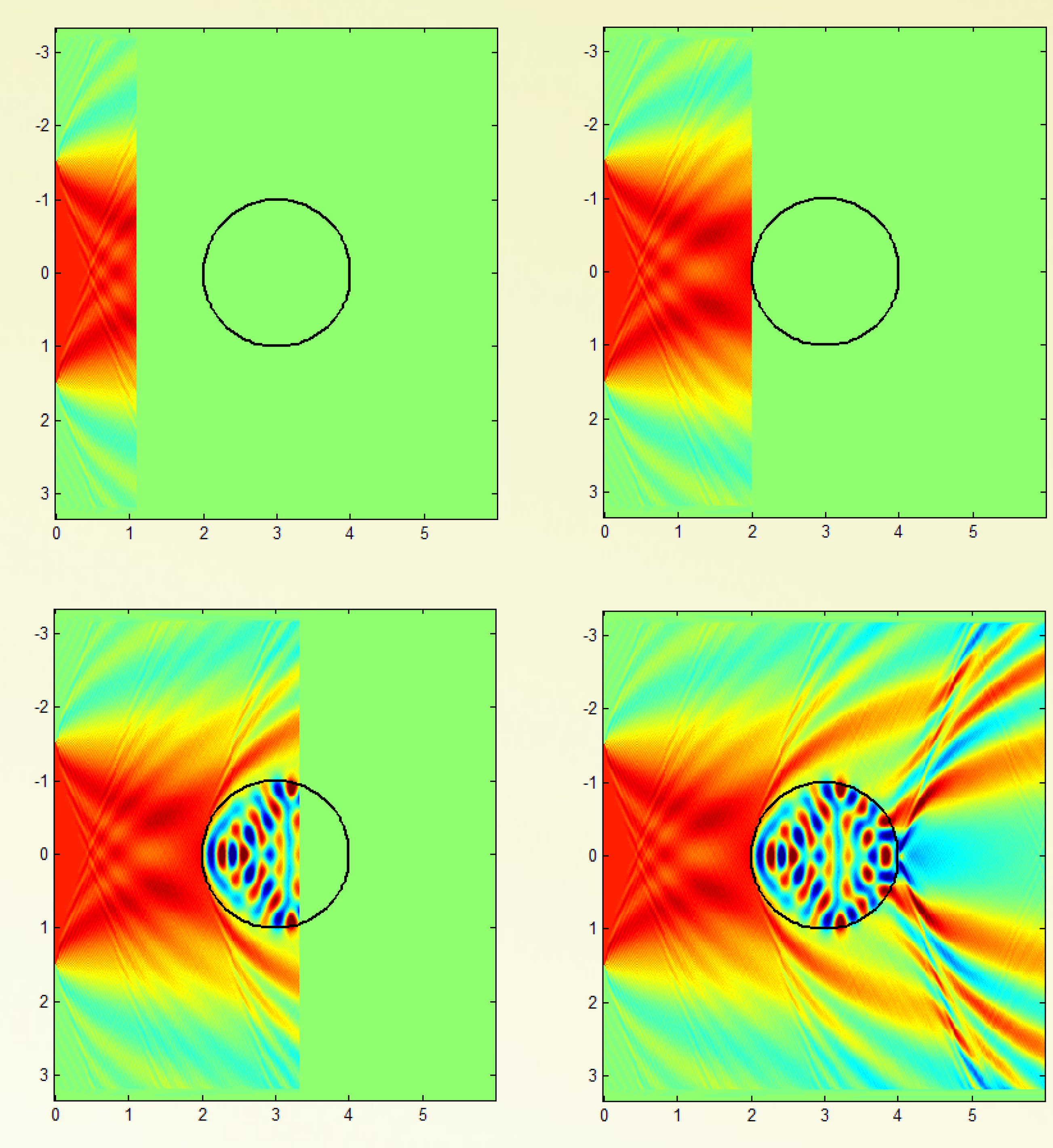
H-Mode

$$\mathbf{h}_y^{i+1}=\left(\mathbf{I}-\frac{j\Delta z'}{4n_{\text{eff}}}\mathbf{A}_h^{i+1}\right)^{-1}\left(\mathbf{I}+\frac{j\Delta z'}{4n_{\text{eff}}}\mathbf{A}_h^i\right)\mathbf{h}_y^i$$

BLOCK DIAGRAM OF THE ALGORITHM



SNAPSHOTS FROM A TYPICAL MODEL



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