Fast Analysis of All-Dielectric Structures Using Rigorous Coupled-Wave Analysis

Rigorous coupled-wave analysis (RCWA) is an extremely efficient algorithm for modeling scattering through all-dielectric structures. It is most efficient for devices with low to moderate dielectric contrast.

**Semi-analytical formulation of Maxwell’s Equations**

Wave Equation for $i^{th}$ Layer

$$\frac{d^2}{dz^2} \mathbf{\Psi}_i(z) - \mathbf{\Omega}_i^2 \mathbf{\Psi}_i(z) = 0$$

$$\mathbf{\Omega}_i = \mathbf{P}_i \mathbf{Q}_i \mathbf{R}_i$$

<table>
<thead>
<tr>
<th>Wave Equation</th>
<th>Solution</th>
<th>K Matrices</th>
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<tbody>
<tr>
<td>$\mathbf{\Psi}_i(z)$</td>
<td>$\mathbf{W}^i = \text{eigen-vector matrix}$</td>
<td>$\lambda^i = \text{eigen-value matrix}$</td>
</tr>
<tr>
<td>$\mathbf{W}^i$</td>
<td>$\mathbf{V} = \text{QW}^i$</td>
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</tr>
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</table>

Plane Wave Amplitudes

$$\mathbf{v}(z) = \mathbf{W}^i \mathbf{u}(z)$$

**The Convolution Matrices**

$$\mathbf{S} = \mathbf{S}_1 \mathbf{S}_2 \mathbf{S}_3 \mathbf{S}_4 \mathbf{S}_5$$

**The scattering matrix**

The formulation for the general asymmetric case is

$$\mathbf{S} = \frac{\mathbf{W}^i}{\mathbf{V}^i}$$

An improved formulation offers the following benefits:

1. Symmetric scattering matrices
2. Faster the calculate.
3. More memory efficient.
4. Interchangeable scattering matrices.

**The scattering matrix algorithm**

Redheffer Star Product:

$$\mathbf{S} = \mathbf{S}_\text{global} = \mathbf{S}_1 \mathbf{S}_2 \mathbf{S}_3 \mathbf{S}_4 \mathbf{S}_5$$

$$\mathbf{S}_1 = a_{i1} \mathbf{S}_i = a_{i1} \mathbf{S}_i$$

$$\mathbf{S}_3 = b_{i1} \mathbf{S}_i = b_{i1} \mathbf{S}_i$$

$$\mathbf{S}_4 = b_{i1} \mathbf{S}_i = b_{i1} \mathbf{S}_i$$

$$\mathbf{S}_5 = b_{i1} \mathbf{S}_i = b_{i1} \mathbf{S}_i$$