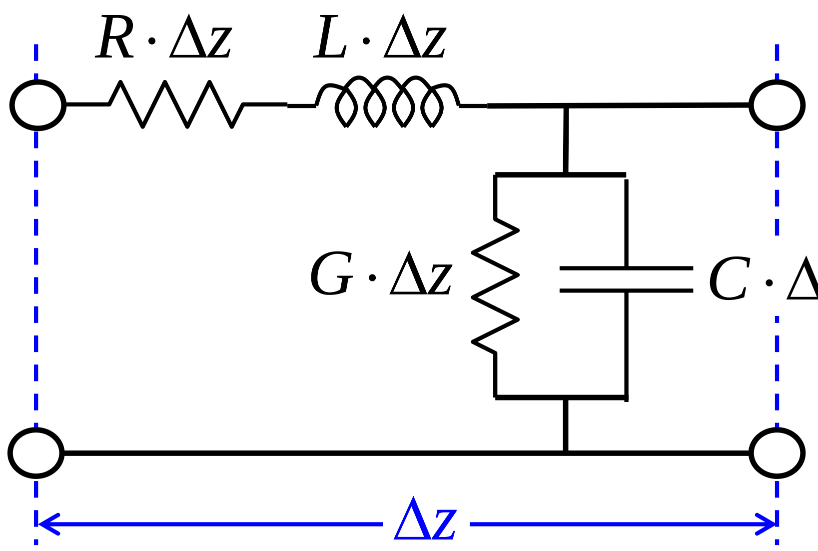
TRANSMISSION LINES

EE 4347 Applied Electromagnetics

Pioneering 21st Century Electromagnetics and Photonics

http://emlab.utep.edu/

Transmission Line Model



$\lim_{\Delta z\to 0}$

$R \equiv$ distributed resistance (Ω/m)
$L \equiv$ distributed inductance (H/m)
$G \equiv$ distributed conductance (1/Ω·m)
$C \equiv$ distributed capacitance (F/m)

Transmission Line Equations

$$-\frac{\partial V(z,t)}{\partial z} = R\cdot I(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

$$-\frac{\partial I(z,t)}{\partial z} = G\cdot V(z,t) + C\frac{\partial V(z,t)}{\partial t}$$

Transmission Line Wave Equations

$$\frac{d^2V(z)}{dz^2} - (R+j\omega L)(G+j\omega C)V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - (R+j\omega L)(G+j\omega C)I(z) = 0$$

Solutions to TL Wave Equations

$$I(z) = I_0^+e^{-\gamma z} + I_0^-e^{\gamma z}$$

$$V(z) = V_0^+e^{-\gamma z} + V_0^-e^{\gamma z}$$

Line Parameters

Characteristic Impedance

$$Z_0 = \frac{V_0^+}{I_0^+} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = R_0 + jX_0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\alpha = \sqrt{\frac{RG-\omega^2LC+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}{2}}$$

$$\beta = \sqrt{\frac{-RG+\omega^2LC+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}{2}}$$

Lossless Lines ($R=G=0$)

$$\alpha = 0 \qquad \beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{L}{C}} \qquad R_0 = \sqrt{\frac{L}{C}} \qquad X_0 = 0$$

Distortionless Lines ($RC=LG$)

$$\alpha = \sqrt{RG} \qquad \beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \qquad R_0 = Z_0 \qquad X_0 = 0$$

Misc.

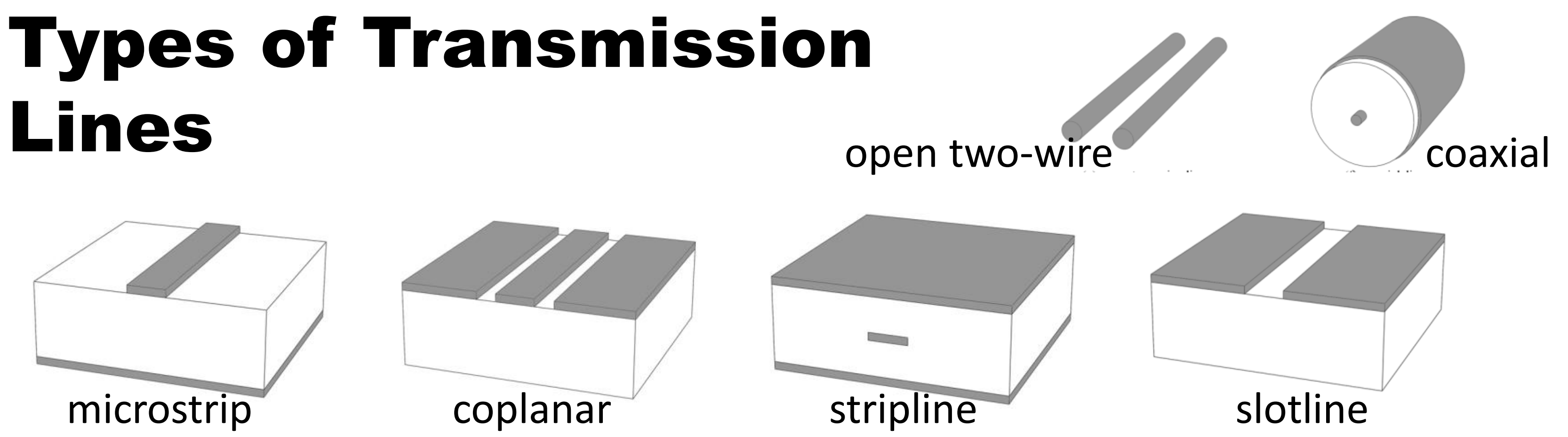
$$P_{\text{avg}} = \frac{|V_0^+|}{2Z_0}\left(1-|\Gamma_L|^2\right)$$

$$Z_{\text{in,short}}Z_{\text{in,open}} = Z_0^2$$

Transmission Line Behavior

General	Short	Open	Matched
Z_L	$Z_L = 0$	$Z_L = \infty$	$Z_L = Z_0$
$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$	$\Gamma_L = -1$	$\Gamma_L = 1$	$\Gamma_L = 0$
Lossy $Z_{\text{in}}(\ell) = Z_0\left(\frac{Z_L + Z_0\tanh\gamma\ell}{Z_0 + Z_L\tanh\gamma\ell}\right)$	Lossy $Z_{\text{in}} = Z_0\tanh\gamma\ell$	Lossy $Z_{\text{in}} = Z_0\coth\gamma\ell$	Lossy $Z_{\text{in}} = Z_0$
Lossless $Z_{\text{in}}(\ell) = Z_0\left(\frac{Z_L + jZ_0\tan\beta\ell}{Z_0 + jZ_L\tan\beta\ell}\right)$	Lossless $Z_{\text{in}} = jZ_0\tan\beta\ell$	Lossless $Z_{\text{in}} = -jZ_0\cot\beta\ell$	Lossless $Z_{\text{in}} = Z_0$
$V_{\max} = \lvert V_0^+\rvert(1+\lvert\Gamma\rvert)$	$V_{\max} = 2\lvert V_0^+\rvert$	$V_{\max} = 2\lvert V_0^+\rvert$	$V_{\max} = \lvert V_0^+\rvert$
$V_{\min} = \lvert V_0^+\rvert(1-\lvert\Gamma\rvert)$	$V_{\min} = 0$	$V_{\min} = 0$	$V_{\min} = \lvert V_0^+\rvert$
$I_{\max} = (\lvert V_0^+\rvert/Z_0)(1+\lvert\Gamma\rvert)$	$I_{\max} = 2\lvert V_0^+\rvert/Z_0$	$I_{\max} = 2\lvert V_0^+\rvert/Z_0$	$I_{\max} = \lvert V_0^+\rvert/Z_0$
$I_{\min} = (\lvert V_0^+\rvert/Z_0)(1-\lvert\Gamma\rvert)$	$I_{\min} = 0$	$I_{\min} = 0$	$I_{\min} = \lvert V_0^+\rvert/Z_0$
$\text{VSWR} = \frac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}$	$\text{VSWR} = \infty$	$\text{VSWR} = \infty$	$\text{VSWR} = 1$
$\max[Z_{\text{in}}] = Z_0\cdot\text{VSWR}$	$\max[Z_{\text{in}}] = \infty$	$\max[Z_{\text{in}}] = \infty$	$\max[Z_{\text{in}}] = Z_0$
$\min[Z_{\text{in}}] = Z_0/\text{VSWR}$	$\min[Z_{\text{in}}] = 0$	$\min[Z_{\text{in}}] = 0$	$\min[Z_{\text{in}}] = Z_0$

Types of Transmission Lines



Impedance Transformation

General Case (with Loss)

$$Z_{\text{in}}(\ell) = Z_0\left(\frac{Z_L + Z_0\tanh\gamma\ell}{Z_0 + Z_L\tanh\gamma\ell}\right)$$

Lossless Case

$$Z_{\text{in}}(\ell) = Z_0\left(\frac{Z_L + jZ_0\tan\beta\ell}{Z_0 + jZ_L\tan\beta\ell}\right)$$