



VECTOR CALCULUS

EE 3321 Electromagnetic Field Theory

Pioneering 21st Century
Electromagnetics and Photonics



<http://emlab.utep.edu>

Dot Products

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

$$\vec{A} \cdot \vec{A} = |\vec{A}|^2$$

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$\nabla \cdot (f\vec{A}) = (\nabla \cdot \vec{A})f + \vec{A} \cdot (\nabla f)$$

Cross Products

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta_{AB} \hat{a}_n$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{A} = 0$$

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

$$\nabla \times (f\vec{A}) = (\nabla \times \vec{A})f + (\nabla f) \times \vec{A}$$

Triple Products

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

Miscellaneous

Stoke's Theorem

$$\oint_L \vec{A} \cdot d\vec{\ell} = \iint_S (\nabla \times \vec{A}) \cdot d\vec{s}$$

Divergence Theorem

$$\oiint_S \vec{A} \cdot d\vec{s} = \iiint_V (\nabla \cdot \vec{A}) dv$$

Cartesian (x, y, z)

Vector Notation

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

Differential Terms

$$d\vec{\ell} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$$

$$d\vec{s} = dydz \hat{a}_x + dx dz \hat{a}_y + dx dy \hat{a}_z$$

$$dv = dx dy dz$$

Vector Derivatives

$$\nabla = \frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial y} \hat{a}_y + \frac{\partial}{\partial z} \hat{a}_z$$

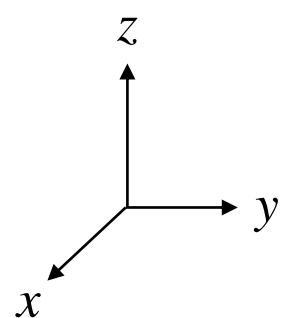
$$\nabla f = \frac{\partial f}{\partial x} \hat{a}_x + \frac{\partial f}{\partial y} \hat{a}_y + \frac{\partial f}{\partial z} \hat{a}_z$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \vec{A} = \nabla^2 A_x \hat{a}_x + \nabla^2 A_y \hat{a}_y + \nabla^2 A_z \hat{a}_z$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \hat{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{a}_z$$



Cylindrical (ρ, φ, z)

Vector Notation

$$\vec{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

Differential Terms

$$d\vec{\ell} = d\rho \hat{a}_\rho + \rho d\phi \hat{a}_\phi + dz \hat{a}_z$$

$$d\vec{s} = \rho d\phi dz \hat{a}_\rho + d\rho dz \hat{a}_\phi + \rho d\rho d\phi \hat{a}_z$$

$$dv = \rho d\rho d\phi dz$$

Vector Derivatives

$$\nabla = \frac{\partial}{\partial \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} \hat{a}_\phi + \frac{\partial}{\partial z} \hat{a}_z$$

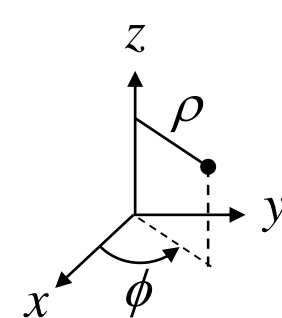
$$\nabla f = \frac{\partial f}{\partial \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{a}_\phi + \frac{\partial f}{\partial z} \hat{a}_z$$

$$\nabla^2 f = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \vec{A} = \nabla^2 A_\rho \hat{a}_\rho + \nabla^2 A_\phi \hat{a}_\phi + \nabla^2 A_z \hat{a}_z$$

$$\nabla \cdot \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{a}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) \hat{a}_\phi + \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho A_\rho) - \frac{\partial A_\rho}{\partial \phi} \right] \hat{a}_z$$



Spherical (r, θ, φ)

Vector Notation

$$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$$

Differential Terms

$$d\vec{\ell} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin \theta d\phi \hat{a}_\phi$$

$$d\vec{s} = r^2 \sin \theta d\theta d\phi \hat{a}_r + r \sin \theta dr d\phi \hat{a}_\theta + r dr d\theta \hat{a}_\phi$$

$$dv = r^2 \sin \theta dr d\theta d\phi$$

Vector Derivatives

$$\nabla = \frac{\partial}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \hat{a}_\phi$$

$$\nabla f = \frac{\partial f}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{a}_\phi$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

$$\nabla^2 \vec{A} = \nabla^2 A_r \hat{a}_r + \nabla^2 A_\theta \hat{a}_\theta + \nabla^2 A_\phi \hat{a}_\phi$$

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right] \hat{a}_r + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right] \hat{a}_\theta + \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \hat{a}_\phi$$

