Problem #1: Build a Dartboard on a Cartesian Grid

The standard English dartboard is shown below. It is composed of the numbers 1 to 20 arranged randomly as wedges in a pie so each wedge spans 18°. The inner bullseye is worth 50 points while the outer bullseye is worth 25 points. Within the outermost ring, the number score is doubled. Inside the middle ring, the number score is tripled. The number scores in each of these regions for the 20-point wedge are shown in the figure. The approximate dimensions of the radii describing these regions are listed to the right.

Write a MATLAB program to build a standard English dartboard onto a Cartesian grid with 512×512 points. This should be a single array with values assigned to each point in a manner that describes the dartboard. You do not have to account for the wires between regions on the board. Image the data in your array to a figure window and include the image in your homework solution. Remember to include your MATLAB code in an Appendix.

Hints:

1. Use `meshgrid()`.
2. Use `atan2()`.
3. Your data should look like the figure to the right.
Problem #2: Calculate Where to Aim

An engineer has just started throwing darts as a hobby and they are not that good yet! After throwing a hundred darts at a paper target, the engineer fit their throwing accuracy to the following two-dimensional Gaussian distribution.

\[
G(x, y) = \frac{1}{2\pi \sigma_x \sigma_y} \exp \left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right)
\]

\[
\sigma_x = 2.12 \text{ inches}
\]
\[
\sigma_y = 3.54 \text{ inches}
\]

Where should this engineer aim on the dartboard to average the highest possible score? What will their average score be if they throw three darts at this position and add the points from all three darts? Justify your answer. Provide a professional plot of \(G(x,y)\). In a second figure, plot the predicted average score (i.e. expectation) throughout the dartboard for a single dart. Mark on this second figure the best point for the thrower to aim.

Hints:
1. It is best to normalize \(G(x,y)\) numerically as follows: \( G = G / \text{sum}(G(:)) \);
2. Think how to use 2D convolutions to solve this problem very simply.
3. Use \texttt{fft2()} to calculate the convolutions.